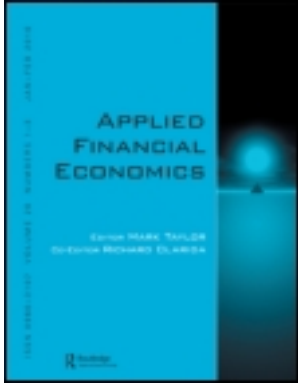


This article was downloaded by: [Lancaster University Library]

On: 15 April 2013, At: 03:46

Publisher: Routledge

Informa Ltd Registered in England and Wales Registered Number: 1072954 Registered office: Mortimer House, 37-41 Mortimer Street, London W1T 3JH, UK



Applied Financial Economics

Publication details, including instructions for authors and subscription information:
<http://www.tandfonline.com/loi/rafe20>

Linkages between Shanghai and Hong Kong stock indices

Shenqiu Zhang^a, Ivan Paya^a & David Peel^a

^a Department of Economics, Lancaster University Management School, Lancaster LA1 4YX, UK

Version of record first published: 23 Oct 2009.

To cite this article: Shenqiu Zhang, Ivan Paya & David Peel (2009): Linkages between Shanghai and Hong Kong stock indices, *Applied Financial Economics*, 19:23, 1847-1857

To link to this article: <http://dx.doi.org/10.1080/09603100903085066>

PLEASE SCROLL DOWN FOR ARTICLE

Full terms and conditions of use: <http://www.tandfonline.com/page/terms-and-conditions>

This article may be used for research, teaching, and private study purposes. Any substantial or systematic reproduction, redistribution, reselling, loan, sub-licensing, systematic supply, or distribution in any form to anyone is expressly forbidden.

The publisher does not give any warranty express or implied or make any representation that the contents will be complete or accurate or up to date. The accuracy of any instructions, formulae, and drug doses should be independently verified with primary sources. The publisher shall not be liable for any loss, actions, claims, proceedings, demand, or costs or damages whatsoever or howsoever caused arising directly or indirectly in connection with or arising out of the use of this material.

Linkages between Shanghai and Hong Kong stock indices

Shenqiu Zhang, Ivan Paya and David Peel*

*Department of Economics, Lancaster University Management School,
Lancaster LA1 4YX, UK*

This article examines the dynamics of the linkages between Shanghai and Hong Kong stock indices. While the volatility linkage is analysed by a Multivariate Generalized Autoregressive Conditional Heteroscedasticity (MVGARCH) framework, the dependence of returns is examined by a copula approach. Eight different copula functions are applied in this study including two time-varying ones which capture the dynamics of the linkage. The result shows significant tail dependence of the returns in the two markets.

I. Introduction

Linkages between international asset returns are important for fund managers in order to diversify risk (Longin and Solnik, 1995), and also for policymakers to monitor the potential for financial contagion (Bae *et al.*, 2003). There is a widespread agreement that international equity markets are linked to one another.¹ However, the strength, type and regional characteristics of those links are still under scrutiny. Recent evidence of financial contagion suggests that this effect appears more pronounced in developing markets, such as Latin America and Asia (Bae *et al.*, 2003).²

The staggering increase in market capitalization of the Chinese stock market in the last 15 years³ has encouraged the analysis of this market and its links with other developed and developing markets.⁴

Poon and Fung (2000) found significant volatility spillovers between Shanghai and Hong Kong by fitting an Autoregressive, AR(1)-Exponential Generalized Autoregressive Conditional Heteroscedasticity, EGARCH(1,1) model with lagged values of the other indices returns and volatilities in the mean and variance equations. A similar result was found by Li (2007) employing a Multivariate GARCH (MVGARCH) model.

*Corresponding author. E-mail: d.peel@lancaster.ac.uk

¹ For a survey of methods and results on the topic, see Heimonen (2002).

² The methodology used in the majority of the papers in this area is the conditional volatility GARCH framework introduced by Engle and Kroner (1995), Engle (2002) and Tse and Tsui (2002). Worthington and Higgs (2004) reported volatility spillovers from Hong Kong and Singapore to Thailand, and also from Japan to Indonesia, Korea, the Philippines and Thailand. However, it is worth pointing out that lagged domestic volatility had a stronger effect on current domestic volatility than the spillover effect. In a study on Central European countries, Kasch-Haroutounian and Price (2001) found spillover effects from the Hungarian equity market to the Polish market, but not the other way round.

³ The stock market valuation of the Chinese equity market was 260 billion yuan in 1994 and 27 trillion yuan in 2008.

⁴ Studies on the Chinese market have found evidence of univariate GARCH(1,1) processes (Yu, 1996; Xu, 1999). However, in the case of a price change limit in the stock market, Friedmann and Sanddorf-Kohle (2002) claimed that the most appropriate model is the MA(1)-Glosten, Jagannathan and Runkle, GJR-GARCH(1,1). Bailey (1994) used single linear equations to investigate linkages between eight individual stocks of the Chinese market. She found little or no correlation with China related shares in Hong Kong and the US stock exchanges. Ma (1996) extended the dataset and analysed the market return within the Capital Asset Pricing Model (CAPM) framework. Ma found additional evidence of correlation with an international risk factor measured by the international beta from the CAPM model.

Li (2007) investigated the relationship of volatility between China, Hong Kong and the US using daily data from January 2001 to August 2005 and found spillover effects from Hong Kong to Shanghai, but not between China and the US.

However, the results of Li (2007) employing the MVGARCH framework have the limitation of assuming equal weight for small and large returns. This specification will not appropriately capture the differential impact, if it occurs, of abnormal movements due to panic selling which may cause large cross border co-movement (Longin and Solnik, 2001).⁵ A method that is able to capture such behaviour is copulas. These have been widely used in many disciplines, such as survival analysis and hydrology (Genest and Favre, 2007) as well as in many studies that examine the correlation between variables. Nonetheless, it is probably the use in finance that has accelerated the development of this methodology. Copulas started to be used in risk management, such as credit risk applications and option pricing (Cherubini *et al.*, 2004), and now they are used in studying market co-movement and financial contagions (Rodriguez, 2007).⁶

In this article we extend the analysis of the linkages between the Shanghai and the Hong Kong equity markets in two ways. First, we employ a longer data set than previous studies in order to capture the recent falls in the Chinese market. Second, we analyse the dependence between the two markets employing both MVGARCH models and a wide variety of different copulas that allow flexible tail behaviour.⁷ Our results suggest the lack of volatility spillover effects using the MVGARCH model. However, the time-varying Symmetrized Joe–Clayton (SJC) copula provided evidence of tail dependence between the Shanghai and the Hong Kong return series.

The rest of this article is organized as follows. Section II will discuss the MVGARCH model. Section III will present the copulas and the methodology used for modelling marginal distributions. Data and its summary statistics will be presented in Section IV. Section V will discuss the empirical results and its implications. Section VI will provide a brief conclusion.

⁵ In a recent study, Longin and Solnik (2001) applied extreme value theory where a multivariate distribution of stock returns tails was proposed and tested. Using monthly stock indices of the US, the UK, France, Germany and Japan from 1959 to 1996, they found that the correlation of returns increases during bear markets.

⁶ Bartram *et al.* (2007) used copulas to examine the effect of the euro on the dependence between European stock indices. For a survey of the copula method in finance, see Patton (2008).

⁷ The only study we are aware of that employs copulas in the analysis of the Chinese stock markets is Ane *et al.* (2008). They studied the relationship between the Chinese markets of Shanghai and Shenzhen exchanges using the Cook–Johnson (Clayton) copula over the period from January 1996 to December 2003. They found persistent features of dependence between Shanghai and Shenzhen stock returns.

II. Multivariate GARCH

We analyse the returns, R_t , of Shanghai and Hong Kong stock markets which are defined as the first difference of the natural logarithm of each stock index. We initially employ the MVGARCH model proposed by Engle and Kroner (1995) where the mean equation is specified as follows:

$$R_t = C + \Theta R'_{t-1} + e_t, \quad e_t \sim N(0, H_t) \quad (1)$$

where R_t is a 2×1 vector. R_{11} is the Shanghai return series, and R_{21} is the Hong Kong return series. H_t is the following variance–covariance matrix:

$$H_t = \Omega\Omega' + A(e_{t-1}e'_{t-1})A' + BH_{t-1}B' \quad (2)$$

In the setting of the Baba, Engle, Kraft and Kroner (BEKK) model, H_t is guaranteed to be positive definite by construction. The conditional variance is not only a function of all lagged conditional variances and squared returns, but also a function of conditional covariances and cross-product returns. The diagonal elements in the parameter matrix B measure the effect of lagged volatility, the off-diagonal elements capture the cross market effects.

III. Copula

Copula functions

A copula is a function that constructs a joint distribution from n marginal distributions. In other words, the copula contains all the dependence information between marginal distributions. The dependence information is only available in a copula, and not in the marginal distribution. This is proved by Sklar in 1959 (Cherubini *et al.*, 2004) (Appendix A). Sklar's theorem allows us to take advantage of flexible univariate modelling methods to obtain dependence information in a multivariate distribution.

We consider six bi-variate copula functions that have different dependence features. These functions are described in detail in Appendix B. The first two copula functions assemble normal distributions and do not consider the possibility of tail dependence. However, since financial time series usually have fat

tails, the last four copula functions under consideration exhibit tail dependence features that measure the joint probability of extreme events.⁸ In this case, it is possible to capture, for instance, greater correlation for large price movements than for small price movements. If the return of an investment is assumed to be a white noise process, small movements represent low or no risk. However, when there are large movements in returns, investors will be alerted to the risk, and some may be interested in hedging those risks.

Copula estimation

All the different types of copula functions considered here can be estimated by the two-stage maximum likelihood method proposed by Patton (2006a). The first stage involves the estimation of the two marginal distributions separately as described in the section below. The conditional marginal distributions are then used in the second stage of the methodology to estimate the copula function. In this case, the log-likelihood function is,

$$L(\theta) = \sum_{t=1}^T \ln C_t(u, v|w) \tag{3}$$

where u and v are the marginal distributions defined in Appendix A.

Conditional marginal models. Conditional volatility GARCH models have been widely used to estimate return series as they can capture stylized facts such as volatility clustering (Berkowitz and O'Brien, 2002). The GARCH family of models has been extended to capture very persistent (or integrated) volatility with the Integrated GARCH (IGARCH) model. To relax such a restrictive case, Equation 4 presents a third type of model that allows for volatility to be fractionally integrated (Fractional Integrated GARCH (FIGARCH)),

$$h_t = \omega + \phi h_{t-1} + \varphi(1 - L)^d \varepsilon_t^2 \tag{4}$$

⁸ The dependence is measured by Spearman's ρ or Kendall's τ statistics. Let us consider $u = F_1(x)$, and $v = F_2(y)$ to be the two marginal distributions. Note that the Gumbel copula exhibits upper tail dependence and zero lower tail dependence, while the Clayton copula has lower tail dependence and zero upper tail dependence. However, either the Student's t and the SJC copulas exhibit both tails dependence.

⁹ The skewed Student's t distribution is more flexible than Student's t distribution. Student's t density is the special case of skewed Student's t when $\lambda = 0$ (Equation 6).

¹⁰ The KS test is a nonparametric statistic that tests if a set of data comes from the hypothesized continuous distribution. Thus, the test has the null hypothesis H_0 : the data follows the specified distribution; and H_a : the data does not follow the specified distribution. The test statistic is,

$$D = \max_{1 \leq i \leq N} \left(F(Y_i) - \frac{i-1}{N}, \frac{i}{N} - F(Y_i) \right)$$

where $F(Y_i)$ is the theoretical cumulative distribution of the specified distribution. The statistics D is compared with tabulated critical values.

where $0 \leq d \leq 1$, $d=0$ represents the GARCH model and $d=1$ the IGARCH model. Given that the persistence of volatility is a stylized fact in financial series (Taylor, 1986), and especially in emerging markets as shown by Ane *et al.* (2008), we specify an AR(1)-FIGARCH-m as the mean equation

$$r_t = \mu + \alpha r_{t-1} + \beta \sqrt{h_t} + \varepsilon_t \tag{5}$$

We specify Hansen (1994) skewed- t distribution for the likelihood estimation, in order to capture skewness and leptokurtosis in the data.⁹ The density function takes the following form:

$$f(z|\eta, \lambda) = \begin{cases} bc \left(1 + \frac{1}{\eta-2} \left(\frac{bz+a}{1-\lambda} \right)^2 \right)^{-(\eta+1)/2} & z < -a/b \\ bc \left(1 + \frac{1}{\eta-2} \left(\frac{bz+a}{1+\lambda} \right)^2 \right)^{-(\eta+1)/2} & z \geq -a/b \end{cases} \tag{6}$$

where $2 < \eta < \infty$ and $-1 < \lambda < 1$, z is the standardized residual and the constants a , b and c are given by

$$\begin{aligned} a &= 4\lambda c \left(\frac{\eta-2}{\eta-1} \right) \\ b^2 &= 1 + 3\lambda^2 - a^2 \\ c &= \frac{\Gamma(\frac{\eta+1}{2})}{\sqrt{\pi(\eta-2)}\Gamma(\frac{\eta}{2})} \end{aligned}$$

The models are estimated by maximizing the following log-likelihood function of the skewed Student's t distribution:

$$\begin{aligned} l = & n \log \Gamma \left(\frac{\eta+1}{2} \right) - \frac{n}{2} \log(\eta) - n \log \left(\Gamma \left(\frac{\eta}{2} \right) \right) - n \log(\sigma) \\ & - \frac{\eta+1}{2} \sum_{i=1}^n \log \left(1 + \frac{\varepsilon_i^2}{\eta\sigma^2} \right) + \sum_{i=1}^n \log(f(z_i|\eta, \lambda)) \end{aligned} \tag{7}$$

The accuracy of the result obtained from a likelihood estimation relies heavily on the specification of the density function, and in order to test that the density is correctly specified, we employ Kolmogorov-Smirnov (KS) test.¹⁰

Table 1. Summary statistics

	Mean	SD	Skewness	Kurtosis	AR(1) ^a	AR(2) ^a
Shanghai	0.04	1.82	-0.21	7.81	0.009	0.005
Hong Kong	0.01	1.78	0.51	16.04	0.008	0.040

Notes: The table shows summary statistics of the returns of the Shanghai and Hong Kong stock market indices. Indices are denominated in Chinese yuan. The sample period covers 1 June 1996 to 1 October 2008 and has 2878 daily observations in each series excluding holidays.

^aAR(*i*) represents the *i*-th lag autocorrelation coefficient of returns.

IV. The Data

Daily data for the Shanghai and Hong Kong composite stock indices are obtained from the Thomson One Banker database for the period 1 June 1996 to 1 October 2008. We use both indices denominated in Chinese yuan. Observations in both series are removed if there is a missing value in one of the series due to holidays, as in Li (2007). As of January 2008, the Shanghai stock exchange is the largest exchange in the mainland with a market value of 22 trillion yuan and 5.6 trillion of tradable volume, while the Shenzhen exchange has a market value of 5 trillion and 2.5 trillion in tradable volume. These figures are based on A-shares which were initially only available to domestic investors but since the end of 2002 are also open to foreign investors through the Qualified Foreign Institutional Investor (QFII) scheme. The other class of shares are called B-shares. These shares are also denominated in yuan, but subscribed to trade in either US dollars or Hong Kong dollars. B-shares were only available to foreign investors before 2001, but since then domestic investors who have US dollars or Hong Kong dollars can trade B-shares as well.

The summary statistics of the returns, defined by changes in the logarithms of these indices times 100, are shown in Table 1. The Shanghai series is negatively skewed, while the Hong Kong series is positively skewed. Both are leptokurtic and have a very low first-order autocorrelation coefficient. Figures 1 and 2 plot the Shanghai and Hong Kong return series and it becomes apparent that the Shanghai index has more outliers than the Hong Kong counterpart.

V. Estimation Results

MVGARCH model

We specify the mean and the variance equations as in Equations 1 and 2 for the MVGARCH model.

¹¹ Initial values of the diagonal parameters are obtained from univariate model estimation and off-diagonal parameter initial values are set to zero.

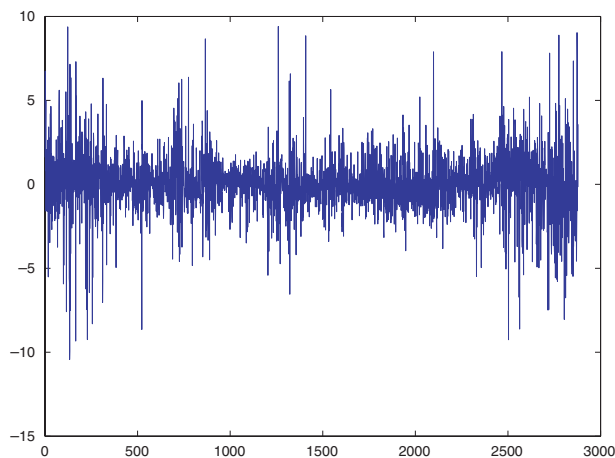


Fig. 1. Shanghai index return from 1 June 1996 to 1 October 2008

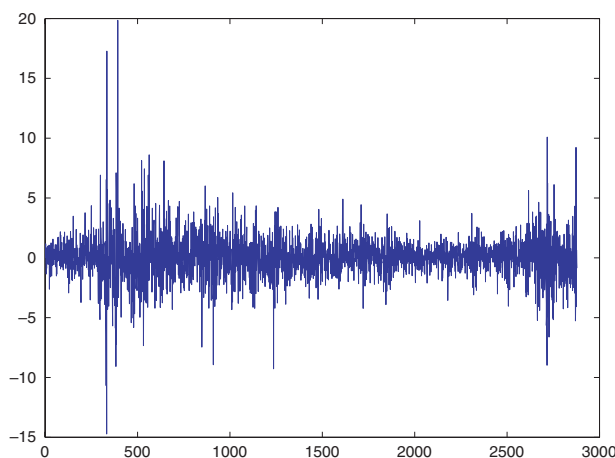


Fig. 2. Hong Kong index return from 1 June 1996 to 1 October 2008

The mean equation includes any possibility of mean return spillovers and we use the joint Student's *t* density functions due to heavy tails in the return series.¹¹ We present the MVGARCH results in Table 2.

Table 2. BEKK estimation

	Shanghai		Hong Kong		
	Estimate	SE	Estimate	SE	
C_{11}	0.039	0.026	C_{21}	0.047*	0.023
Θ_{11}	0.016	0.020	Θ_{21}	-0.007	0.014
Θ_{12}	0.042*	0.015	Θ_{22}	0.005	0.018
Ω_{11}	0.736*	0.040	Ω_{21}	0.503*	0.043
A_{11}	0.301*	0.031	A_{21}	-0.009	0.013
A_{12}	-0.010	0.017	A_{22}	0.192*	0.027
B_{11}	0.933*	0.014	B_{21}	0.007	0.005
B_{12}	0.002	0.004	B_{22}	0.975*	0.006
	Statistic	<i>p</i> -value	Statistic	<i>p</i> -value	
$Q(12)$	48.73	0.00	24.51	0.22	
$Q^2(12)$	7.08	0.93	41.10	0.01	

Note: * Indicates significance at 5% level.

The BEKK model provides evidence of return linkages in the mean equation as Θ_{12} is significant. This result implies return spillovers from Hong Kong to Shanghai, but not from Shanghai to Hong Kong given that Θ_{21} is insignificant. The positive coefficient implies that the return of Hong Kong is transmitted with the same sign to Shanghai in the next day, but not the other way round. It is worth pointing out that the linkage is weak as the spillover is only 4%. On the other hand, we did not find any evidence of volatility linkages between the two markets as the off-diagonal terms in the variance equation are not significant. These conclusions are different from those of Li (2007), who found no linkage in the return series but found volatility spillover from Hong Kong to Shanghai. The difference might arise from the fact that our study uses a more updated data set that includes the most recent stock market crash.

Copula models

Result of conditional marginal models. Table 3 shows the FIGARCH-m results of the two conditional marginal models. Both return series display similar patterns, and the value of the variance equation parameters are very close. The fractional parameter d is significant in both cases with values 0.59 for Shanghai and 0.57 for Hong Kong. The parameter φ is again similar and indicates a strong effect from past squared conditional returns. λ and η are the degrees of freedom and skewness parameters. The estimated value of these two parameters for Shanghai are very

Table 3. Result of conditional marginal models

	Shanghai		Hong Kong	
	Estimate	SE	Estimate	SE
μ	-0.079*	0.041	0.055*	0.024
β	0.042*	0.013	-	-
α	0.009	0.018	0.013	0.018
ω	0.439*	0.086	0.210*	0.054
ϕ	0.136*	0.073	0.158*	0.0482
φ	0.600*	0.131	0.695*	0.070
d	0.599*	0.119	0.574*	0.080
λ	4.138*	0.080	5.567*	0.119
η	-0.058*	0.024	-0.006	0.022
	Statistic	<i>p</i> -value	Statistic	<i>p</i> -value
Box-Pierce	43.167	0.000	21.935	0.292
$Q(12)$				
Box-Pierce	8.040	0.992	16.564	0.681
$Q^2(12)$				
KS test	0.016	0.493	0.008	0.025
Log-likelihood	-5291.61	-	-5102.62	-

Note: * Indicates significance at 5% level.

close to those found in Ane *et al.* (2008). The difference between Shanghai and Hong Kong is that in the latter case returns do not have volatility effects in the mean equation while in Shanghai they do.

Copula estimation. The conditional marginal densities estimated above are now used to estimate the copula functions and Table 4 presents the log-likelihood estimates.¹² Considering first the constant parameter copula function we find that the SJC and the Student's t copulas appear to have the highest log-likelihood figures. Moreover, there seems to be an improvement in the log-likelihood when the parameters in the copula functions are allowed to change over time. In particular, the time-varying SJC copula is significantly¹³ improved over the constant SJC copula and implies that the dependence between the two markets changes over time.

To further examine the tail dependence between markets, Table 5 shows the estimates of the time-varying SJC copula. The coefficients indicate that there is different co-movement at both the lower and upper tails. α and β are -7.18 (-18.28) and -19.45 (-14.27) at the lower(upper) tail and significantly different from zero. The fact that the parameters governing time-varying α and β are greater than the constant parameter ω implies strong time-varying effect. The implied correlation coefficient from this

¹² All copula functions are estimated in Matlab. The authors are grateful to Andrew Patton for making the codes publicly available.

¹³ Indicated by Likelihood Ratio (LR) tests with 6 degrees of freedom at 5% significance level.

Table 4. Log-likelihood of Copula estimation

	Copula	Log-likelihood	AIC	BIC
Constant	SJC	47.4648	-94.9282	-94.9241
	Student <i>t</i>	45.8692	-91.7369	-91.7328
	Gumbel	42.0931	-84.1856	-84.1835
	Gaussian	41.5751	-83.1496	-83.1475
	Plackett	35.0474	-70.0942	-70.0921
	Clayton	32.7873	-65.5740	-65.5719
Time-varying	SJC	55.5064	-111.0086	-110.9962
	Gaussian	49.8858	-99.7695	-99.7633

Note: AIC = Akaike's Information Criterion; BIC = Bayesian Information Criterion.

Table 5. Time-varying SJC copula

Parameter	Coefficient	SE
ω^L	2.590*	1.19*
α^L	-7.18*	3.65*
β^L	-19.44*	5.45*
ω^U	1.51*	0.81*
α^U	-18.28*	7.78*
β^U	-14.27*	3.31*

Notes: SEs are calculated from Hessian matrix.

* Indicates significance at 5% level.

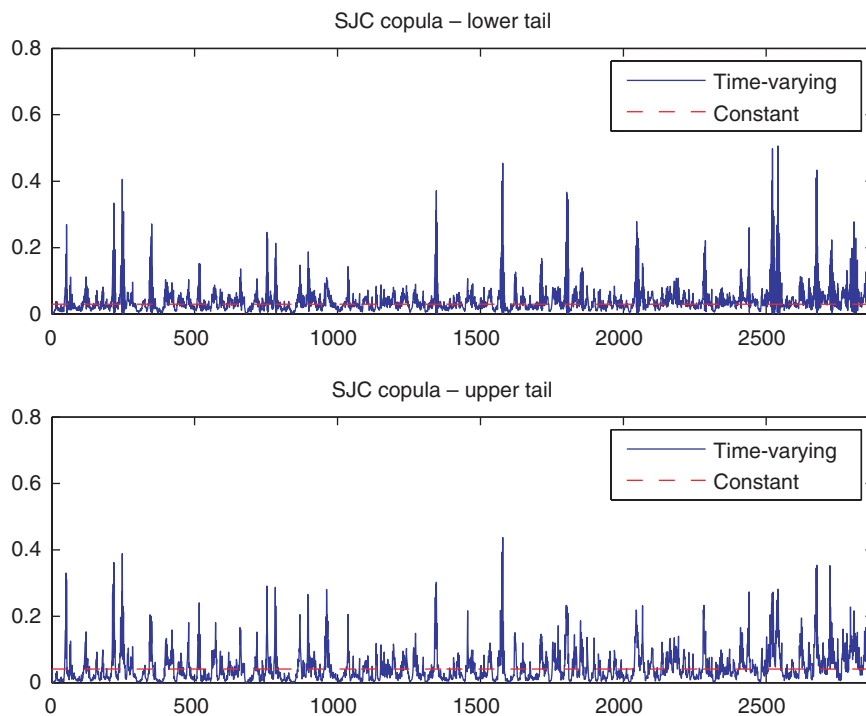


Fig. 3. Correlation implied by time-varying SJC

Table 6. Hong Kong return series in HK dollar and Chinese yuan domination

	Mean	SD	Skewness	Kurtosis	AR(1) ^a	AR(2) ^a
HK dollar	0.02	1.78	0.49	16.09	0.01	0.04
Chinese yuan	0.01	1.78	0.51	16.04	0.01	0.04

Note: ^aAR(*i*) represents the *i*-th lag autocorrelation coefficient of returns.

estimate is shown in Fig. 3 and changes over time, reaching a maximum of 0.5 in some cases. In this sense, the copula approach provides additional information to the MVGARCH methodology by showing the dependence at the tails.

Robust analysis: the exchange rate effect

The data analysed are both denominated in Chinese currency. However, Hong Kong stock markets are traded in Hong Kong dollars. Thus, there exists a possibility of an exchange rate effect. In order to examine such effect, we analyse the Hong Kong series in its native currency. Table 6 compares the summary statistic of Hong Kong index return with different currency denomination and their change. As a further check we estimate the copula functions using the Hong Kong series dominated in Hong Kong dollars and we do not find any significant change of results.¹⁴

VI. Conclusion

In this article we have examined the linkages between the Shanghai and Hong Kong stock indices using two different methodologies. On the one hand, the MVGARCH model results suggest that there are spillover effects in the mean of returns but not in the volatility. The conditional marginal models estimated by FIGARCH with skewed Student's *t* density showed that volatilities of the two indices returns are persistent. However, the parametric GARCH methodology does not allow for the dependence in volatility to differ according to the size and sign of changes in returns. In order to capture such effect, we use six different copula functions and found evidence of volatility linkages at both tails. The SJC copula was found to provide the better fit among all of them and we also estimated it using

time-varying parameters. The results again imply significant tail dependence which has varied over time in the past decade.

Acknowledgements

I. Paya is grateful for financial support from the Spanish Ministerio de Educacion y Ciencia Research Project ECO2008-05721/ECON.

References

- Ane, T., Ureche-Rangau, L. and Labidi-Makni, C. (2008) Time-varying conditional dependence in Chinese stock markets, *Applied Financial Economics*, **18**, 895–916.
- Bae, K.-H., Karolyi, G. A. and Stulz, M. R. (2003) A new approach to measuring financial contagion, *The Review of Financial Studies*, **16**, 717–63.
- Bailey, W. (1994) Risk and return on China's new stock markets: some preliminary evidence, *Pacific-Basin Finance Journal*, **2**, 243–60.
- Bartram, S. M., Taylor, S. J. and Wang, Y.-H. (2007) The euro and European financial market dependence, *Journal of Banking and Finance*, **31**, 1461–81.
- Berkowitz, J. and O'Brien, J. (2002) How accurate are value-at-risk models at commercial banks, *The Journal of Finance*, **57**, 1093–111.
- Busetti, F. and Harvey, A. (2008) When is a copula constant? A test for changing relationships, Cambridge Working Papers in Economics, No. 0841, University of Cambridge.
- Cherubini, U., Luciano, E. and Walter, V. (2004) *Copula Methods in Finance*, John Wiley & Sons, Ltd, Chichester.
- Clayton, D. G. (1978) A model for association in bivariate life tables and its application in epidemiological studies of familial tendency in chronic disease incidence, *Biometrika*, **65**, 141–51.
- Engle, R. (2002) Dynamic conditional correlation: a simple class of multivariate generalized autoregressive conditional heteroskedasticity models, *Journal of Business and Economic Statistics*, **20**, 339–50.
- Engle, R. F. and Kroner, K. F. (1995) Multivariate simultaneous generalized ARCH, *Econometric Theory*, **11**, 122–50.
- Friedmann, R. and Sanddorf-Kohle, W. G. (2002) Volatility clustering and nontrading days in Chinese stock markets, *Journal of Economics and Business*, **54**, 193–217.
- Genest, C. and Favre, A. (2007) Everything you always wanted to know about copula modeling but were afraid to ask, *Journal of Hydrologic Engineering*, **12**, 347–68.
- Hansen, B. E. (1994) Autoregressive conditional density estimation, *International Economic Review*, **35**, 705–30.
- Heimonen, K. (2002) Stock market integration: evidence on price integration and return convergence, *Applied Financial Economics*, **12**, 415–29.

¹⁴ Results are available from the authors upon request.

- Joe, H. (1997) *Multivariate Models and Dependence Concepts*, Chapman & Hall, London.
- Kasch-Haroutounian, M. and Price, S. (2001) Volatility in the transition markets of Central Europe, *Applied Financial Economics*, **11**, 93–105.
- Li, H. (2007) International linkages of the Chinese stock exchanges: a multivariate GARCH analysis, *Applied Financial Economics*, **17**, 285–97.
- Longin, F. and Solnik, B. (1995) Is the correlation in international equity returns constant: 1960–1990?, *Journal of International Money and Finance*, **14**, 3–26.
- Longin, F. and Solnik, B. (2001) Extreme correlation of international equity markets, *The Journal of Finance*, **56**, 649–76.
- Ma, X. (1996) Capital controls, market segmentation and stock prices: evidence from the Chinese stock market, *Pacific-Basin Finance Journal*, **4**, 219–39.
- Nelsen, R. B. (1998) *An Introduction to Copulas*, Lecture Notes in Statistics, Springer, New York.
- Patton, A. J. (2006a) Estimation of multivariate models for time series of possibly different lengths, *Journal of Applied Econometrics*, **21**, 147–73.
- Patton, A. J. (2006b) Modelling asymmetric exchange rate dependence, *International Economic Review*, **47**, 527–56.
- Patton, A. J. (2008) Copula-based models for financial time series, OFRC Working Papers Series No. 2008fe21, Oxford Financial Research Centre.
- Poon, W. P. H. and Fung, H.-G. (2000) Red chips or H shares: which China-backed securities process information the fastest?, *Journal of Multinational Financial Management*, **10**, 315–43.
- Rodriguez, J. C. (2007) Measuring financial contagion: a copula approach, *Journal of Empirical Finance*, **14**, 401–23.
- Sims, C. A. and Zha, T. (2006) Were there regime switches in US monetary policy?, *American Economic Review*, **96**, 54–81.
- Taylor, S. (1986) *Modelling Financial Time Series*, Wiley, Chichester, UK.
- Tse, Y. K. and Tsui, K. C. (2002) A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations, *Journal of Business and Economic Statistics*, **20**, 351–62.
- Worthington, A. and Higgs, H. (2004) Transmission of equity returns and volatility in Asian developed and emerging markets: a multivariate GARCH analysis, *International Journal of Finance and Economics*, **9**, 71–80.
- Xu, J. (1999) Modeling Shanghai stock market volatility, *Annals of Operations Research*, **87**, 141–52.
- Yu, W. (1996) A conditional variance model for daily stock returns in China's emerging stock markets: empirical evidence on the Shanghai and Shenzhen exchanges, *Journal of International Financial Markets, Institutions and Money*, **6**, 1–19.

Appendix A: Sklar's Theorem

Let $F_1(x)$ be the marginal distribution of x , $F_2(y)$ the marginal distribution of y , and $H(x, y)$ the joint distribution of (x, y) . Then for every $(x, y) \in \mathfrak{R} * \mathfrak{R} : C(F_1(x), F_2(y))$ is a joint distribution with margins $F_1(x), F_2(y)$:

$$C(F_1(x), F_2(y)) = H(x, y) \quad (8)$$

Conversely, if $H(x, y)$ is a joint distribution function with margins $F_1(x), F_2(y)$, there exists a copula C , such that

$$H(x, y) = C(F_1(x), F_2(y)) \quad (9)$$

the copula is unique if $F_1(x), F_2(y)$ are continuous, otherwise uniqueness is not guaranteed.

Appendix B: Copula Functions

Gaussian copula

The Gaussian copula describes the bi-normal joint distribution which is the most basic distribution

function in finance.

$$C_{nc}(u, v|\rho) = \frac{1}{\sqrt{1-\rho^2}} \exp\left\{\frac{-(u^2 + v^2 - 2\rho uv)}{2(1-\rho^2)} + \frac{1}{2}(u^2 + v^2)\right\} \quad (10)$$

the parameter ρ is the pairwise correlation. The Gaussian copula was used in Bartram *et al.* (2007) who investigated the dependence in the European financial markets. They found a significant change of dependence structure in European equity market after the introduction of euro.

Plackett copula

The Gaussian copula restricts ρ to be between 0 and 1. Hence it does not allow negative dependence. The Plackett copula (PC) is designed to relax this restriction.

$$C_{pc}(u, v|\theta) = \left\{1 + (\theta - 1)(u + v) - [(1 + (\theta - 1)(u + v))^2 - 4uv\theta(\theta - 1)]^{1/2}\right\} \times [2(\theta - 1)]^{-1} \quad (11)$$

where $\theta > 1$ implies positive dependence, $\theta < 1$ implies negative dependence and $\theta = 1$ implies independence.

The dependence is described by Spearman's ρ as in (Nelsen, 1998),

$$\rho = \begin{cases} \frac{\theta + 1}{\theta - 1} - \frac{2\theta}{(\theta - 1)^2} \ln(\theta) & \text{if } \theta \neq 1 \\ 0 & \text{if } \theta = 1 \end{cases} \quad (12)$$

Gumbel copula

The Gumbel copula exhibits upper tail dependence and zero lower tail dependence.

$$C_{gc}(u, v|\alpha) = \exp\left\{-[(-\ln u)^\alpha + (-\ln v)^\alpha]^{1/\alpha}\right\} \quad (13)$$

The upper tail dependence equals $2 - 2^{1/\alpha}$.

Clayton copula

Introduced by Clayton (1978), this copula is the opposite of the Gumbel copula. It has lower tail dependence and zero upper tail dependence.

$$\begin{aligned} \max[(u^{-\alpha} + v^{-\alpha} - 1)^{-1/\alpha}, 0] \\ \tau = \alpha/(\alpha + 2) \end{aligned} \quad (14)$$

The lower tail dependence equals $2^{-1/\alpha}$.

Student's t copula

The previous two copula functions model one tail behaviour. Student's t copula exhibits dependence in both tails and the dependence is symmetric.

$$\begin{aligned} C_{tc}(u, v) = \frac{\Gamma(\frac{\nu+2}{2})}{\Gamma(\frac{\nu}{2})\nu\pi\sqrt{1-\rho^2}} \\ \times \left(1 + \frac{t^{-1}(u)^2 + t^{-1}(v)^2 - 2\rho t^{-1}(u)t^{-1}(v)}{\nu(1-\rho^2)}\right)^{-\nu+2/2} \end{aligned} \quad (15)$$

where ν is the number of degrees of freedom. If ν becomes large Student's t copula becomes Gaussian copula.

SJC copula

Student's t copula exhibits symmetric dependence in both tails. However, it might be reasonable to assume that the dependence will be different between bear and bull markets. In other words, the dependence could be asymmetric. Joe (1997) introduced the Joe–Clayton copula to capture such

behaviour,

$$\begin{aligned} C_{JC}(u, v|\tau^U, \tau^L) = 1 - (1 - \{[1 - (1 - u)^\kappa]^{-\gamma} \\ + [1 - (1 - v)^\kappa]^{-\gamma} - 1\}^{-1/\gamma})^{1/\kappa} \end{aligned} \quad (16)$$

where $\kappa = 1/\log_2(2 - \tau^U)$, $\gamma = -1/\log_2(\tau^L)$ and $\tau^U \in (0, 1)$, $\tau^L \in (0, 1)$. τ^L and τ^U are the two parameters of this copula that measures lower and upper tail dependence, respectively. One small problem is that when τ^L and τ^U are equal, there is still asymmetry. However, Patton (2006b) modified the Joe–Clayton copula and obtained the SJC copula,

$$\begin{aligned} C_{SJC}(u, v|\tau^U, \tau^L) = 0.5(C_{JC}(u, v|\tau^U, \tau^L) \\ + C_{JC}(1 - u, 1 - v|\tau^L, \tau^U) \\ + u + v - 1) \end{aligned} \quad (17)$$

Copula functions are complex and, therefore, it is not useful to plot a copula function. Instead, we plot the effect of the copula functions, that is, we plot the joint probability distribution against its marginal distributions. We show these plots in Figs A1–A3. The plot shows different dependence structures implied by different copulas. For instance, the Clayton copula is showing the lower tail dependence while the Gumbel copula is showing upper tail dependence. A negative dependence is simulated by the PC. Student's t copula displays the symmetric tail dependence, while the SJC copula can have different tail dependence.

Conditional copula

As previously discussed, the Sklar's theorem separates the joint distribution into n marginal distributions and a copula. This potentially provides the foundation to build on the success of univariate modelling methods to study multivariate distributions. This involves specifying a model for the marginal distributions and a copula. In economics and finance, many series are conditional on some other variables. An extension to conditional distribution of Sklar's theorem is therefore required and this is provided by Patton (2006b).

Let w be an information set, $F_1(x|w)$ the conditional marginal distribution of x conditional on w , $F_2(y|w)$ the conditional marginal distribution of y conditional on w , and $H(x, y|w)$ the joint conditional distribution of (x, y) conditional on w . Then for every $(x, y) \in \Re * \Re : C(F_1(x), F_2(y))$ is a joint distribution with margins $F_1(x), F_2(y)$

$$C(F_1(x|w), F_2(y|w)|w) = H(x, y|w) \quad (18)$$

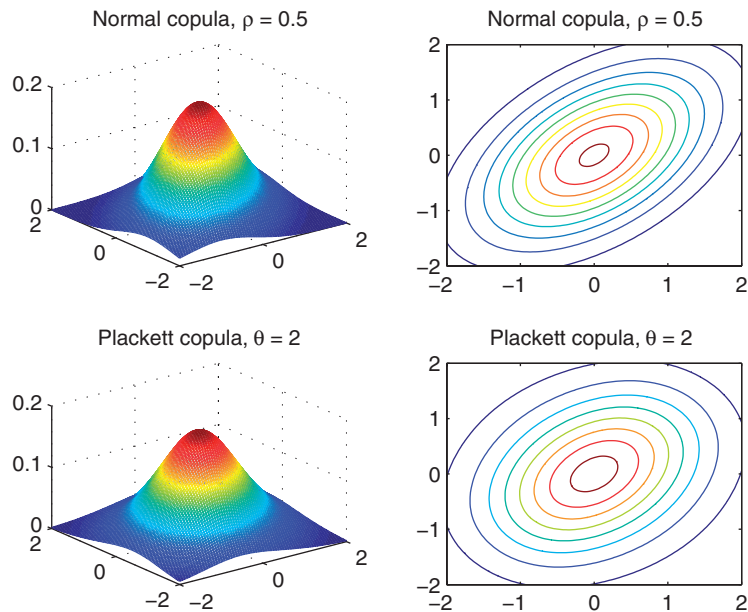


Fig. A1. Copula simulation three-dimensional and contour plot, Normal and Plackett copula

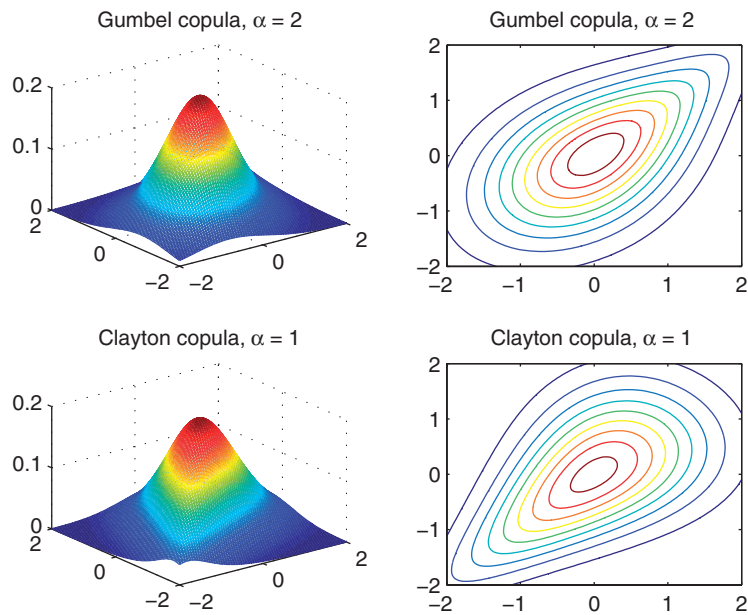


Fig. A2. Copula simulation three-dimensional and contour plot, Gumbel and Clayton copula

conversely, if $H(x, y|w)$ is a joint conditional distribution function with margins $F_1(x|w)$, $F_2(y|w)$, there exists a copula C , such that

$$H(x, y|w) = C(F_1(x|w), F_2(y|w)|w) \quad (19)$$

the copula is unique if $F_1(x|w)$, $F_2(y|w)$ are continuous, otherwise this is not guaranteed. Note that w is the same for both marginal distributions. In other words, the two marginal models need to be

conditional on the same information set. Therefore, in empirical estimation, the same conditional variables should be used. However, conditional variables are allowed to be insignificant.

Time-varying copulas

The dependence structure may change over time due to changes in policy, such as changes in monetary rules (Sims and Zha, 2006) or the introduction of

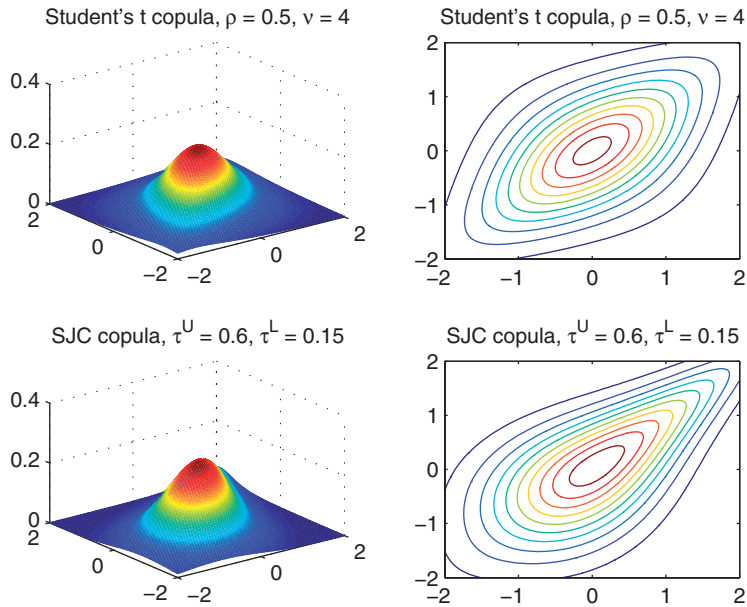


Fig. A3. Copula simulation three-dimensional and contour plot, Student *t* and SJC copula

a common currency such as euro (Patton, 2006b; Bartram *et al.*, 2007). Therefore, the copula may not be constant (Buseti and Harvey, 2008). There are two ways to handle a changing copula. One is to specify a mixture copula function. The difficulty of this option is that with so many copula functions, it is difficult to specify the mixture. The alternative way is to apply a time-varying copula, that is, a copula with time-varying parameters as in Patton (2006b). Equation 20 shows the time evolving parameter for the time-varying normal copula. τ_t^U and τ_t^L in Equations 21 and 22 are the time-varying parameters for time-varying SJC copula:

$$\rho_t = f\left(\omega + \beta\rho_{t-1} + \alpha\frac{1}{n}\sum_{j=1}^n \Phi^{-1}(u_{t-j})\Phi^{-1}(v_{t-j})\right) \quad (20)$$

where $f(x) = \tanh\left(\frac{x}{2}\right) = (1 - e^{-x})(1 + e^{-x})^{-1}$, this transformation will guarantee ρ to be in $(-1, 1)$.

$$\tau_t^U = f\left(\omega^U + \beta^U\tau_{t-1}^U + \alpha^U\frac{1}{n}\sum_{j=1}^n |u_{t-j} - v_{t-j}|\right) \quad (21)$$

$$\tau_t^L = f\left(\omega^L + \beta^L\tau_{t-1}^L + \alpha^L\frac{1}{n}\sum_{j=1}^n |u_{t-j} - v_{t-j}|\right) \quad (22)$$

where $f(x) = (1 + e^{-x})^{-1}$ is the logistic transformation, which will keep τ^U and τ^L in $(0,1)$ at all times. $|u_{t-j} - v_{t-j}|$ is a forcing variable used by Patton (2006b) as an innovation term. Different forcing variables have been tried by Patton (2006b) and Bartram *et al.* (2007), and this is the preferred one.