



# Higher education institutions' costs and efficiency: Taking the decomposition a further step

Geraint Johnes\*, Jill Johnes

Lancaster University Management School, Lancaster LA1 4YX, United Kingdom

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## ABSTRACT

A multiproduct cost function is estimated for English higher education institutions using a panel of data from recent years. The panel approach allows estimation by means of a random parameter stochastic frontier model which provides considerable new insights in that it allows the impact on costs of inter-institutional differences in the cost function itself to be distinguished from inter-institutional differences in efficiency. The approach used here therefore resembles in some respects the non-parametric methods of efficiency evaluation. We report also on measures of average incremental cost of provision and on returns to scale and scope.

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## 1. Introduction

The evaluation of costs in higher education has long been of interest to students of education economics. Theoretical insights into the operation of multiproduct organisations by Baumol, Panzar, and Willig (1982) rapidly led to the empirical evaluation of sophisticated cost functions in the sphere of higher education (Cohn, Rhine, & Santos, 1989). Later contributions refined the analysis by using frontier estimation methods that simultaneously evaluate cost structures and the technical efficiency of institutions (Johnes, 1996).<sup>1</sup> In these papers a parametric cost function is estimated which is assumed to be stable across

institutions that are heterogeneous in terms of their efficiency. The present paper takes the analysis a step further by estimating costs in a framework that allows institutions to differ in terms of both their efficiencies and their cost technologies.

Research on efficiency measurement has, since the seminal work of Farrell (1957) bifurcated, with economists typically following the route of statistical analysis (Aigner, Lovell, & Schmidt, 1977) and management scientists characteristically opting for a non-parametric route grounded in linear programming (Charnes, Cooper, & Rhodes, 1978). The former approach has come to be known as stochastic frontier analysis, the latter as data envelopment analysis (DEA). The relative merits and demerits of the two approaches are by now well known: the parametric statistical approach benefits from the availability of the toolkit of statistical inference, but imposes a common functional form and common parameters on all decision-making units; the alternative non-parametric approach is attractive in that it does not impose a common loss function on all units, but it

\* Corresponding author. Tel.: +44 1524 594215; fax: +44 1524 594244.  
E-mail addresses: [G.Johnes@lancs.ac.uk](mailto:G.Johnes@lancs.ac.uk), [g.johnes@lancs.ac.uk](mailto:g.johnes@lancs.ac.uk)  
(G. Johnes).

<sup>1</sup> Frontier methods of this kind have also been used to examine production functions in education—see, for example, Cooper and Cohn (1997).

lacks a statistical apparatus and its results may be sensitive to the presence of outliers.

Recent developments in the analysis of panel data have made available a new approach which combines the merits of both the statistical and non-parametric methodologies while suffering from none of the drawbacks. Tsionas (2002) and Greene (2005) have developed random parameter formulations of the stochastic frontier model which (in common with DEA) allow a separate loss function to be estimated for each decision-making unit while (in common with traditional frontier models) retaining the apparatus of statistical inference. In essence these models are simply a generalisation of the random effects frontier model introduced by Battese and Coelli (1995); while the random effects model allows only the constant to vary across decision-making units, however, the random parameters model allows any number of the other coefficients to vary as well. A distinction between these models and DEA is that the cross-unit variation is constrained to follow a specified statistical distribution; this constraint allows us to retain the toolkit of statistical inference.

In the context of higher education institutions, the development of this new methodology is particularly significant. It is well understood that HEIs do not represent an homogenous group. Some are old, some are new, some are big, some are small, some focus on certain subject groups, others focus on others, some are comprehensive in their provision, others are more specialised, some are research intensive, others not, and so on. Early studies of cost functions for UK institutions (such as Glass, McKillop, & Hyndman, 1995a, 1995b) focused purely on traditional universities. Later studies (for example, Johnes, 1997) looked at all universities, but excluded other providers of higher education such as colleges. The most recent work (Johnes, Johnes, Thanassoulis, Lenton, & Emrouznejad, 2005) includes higher education colleges as well as universities, but devotes much space to the separate estimation of cost functions specific to certain pre-specified groups of institutions. This approach is far from ideal, however, because the distinctions between traditional universities, former polytechnics, and colleges of higher education have become increasingly blurred over time. An alternative approach, and the one on which the present paper is founded, is to develop an integrated framework for the estimation of costs, but to let the data decide the parameters of the cost function that apply uniquely to each institution.

To motivate the analysis a little further, consider a comparison between four institutions. One is an ancient university, where learning is delivered primarily through small group tutorials. This university has high costs because the student:staff ratio is necessarily low. But it delivers learning in a form that might be deemed desirable, albeit not one that would be cost-effective if applied to the mass of higher education institutions.<sup>2</sup> The second institution might also have high costs, but in this case they are due to locational factors; perhaps the institution is located in

the nation's capital, where space and other costs are relatively high. The third institution has relatively high costs because (within the subject mix categories used in the analysis) it teaches expensive subjects; for instance, medicine may be more costly to deliver than other science subjects, but our analysis fails to disaggregate subjects sufficiently to identify medicine as a separate output. The fourth institution has moderate costs, as it does not have an adverse location or a need to employ unusually expensive teaching technologies. Now in a simple cross-section frontier analysis, the first three institutions may appear to be inefficient because of their high costs. In fact, however, there are reasonable explanations for these high costs, and these should not necessarily be put down to inefficiency. It is clear, therefore, that it is desirable that we should establish a method whereby unobserved heterogeneity in the cost function across institutions, on the one hand, and inefficiency, on the other, can be disentangled. That is the aim of this paper.

We employ recent developments in order to analyse the cost function for each higher education institution in England. Both random effects and more general random parameters models are estimated using panel data for 3 years, 2000–2001 through 2002–2003. Hence differences in intercept and slope coefficients across institutions can be estimated alongside differences in institutions' efficiency. The next section discusses the data. Results and analysis are provided in the following section. The paper ends with a conclusion and suggestions for further research.

## 2. Data

Our data are drawn from English institutions of higher education over a 3-year period from 2000–2001 through 2002–2003. Some 121 institutions are included in the analysis; this includes ancient universities (such as Oxford and Cambridge), traditional universities (comprising all those institutions with university status prior to 1992), new universities (granted university status in or since 1992), and Colleges of Higher Education.<sup>3</sup> The sample therefore includes a heterogeneity of institutional types, and it is likely that it would be inappropriate to impose on any model of costs based on this sample a parametric form that does not allow coefficients to vary at least somewhat across observations.

All data are obtained from the Higher Education Statistics Agency (HESA): aggregate student numbers are published in *Students in Higher Education Institutions*, and financial statistics are available from *Resources in Higher Education Institutions*; institution-specific information about student numbers, disaggregated by subject area, was obtained from unpublished HESA sources. All financial data used in the study have been adjusted to 2002–2003 values.<sup>4</sup>

<sup>2</sup> We realise, of course, that this is contentious. The assumption here is that the user of the analysis has a will to see teaching technologies of this kind preserved in some institutions but not others.

<sup>3</sup> A small number of institutions which changed significantly in character over a 3-year period, and for which therefore consistent data series are not available, is excluded from the sample.

<sup>4</sup> RPI inflators of 1.0366 and 1.0294 were applied to 2000/2001 and 2001/2002 figures, respectively.

**Table 1**  
Descriptive statistics

Variable	Mean	Standard deviation
Costs (£ × 10 <sup>7</sup> , 2003 prices)	8.593	8.990
Science undergraduates ('000)	2.760	2.519
Non-science undergraduates ('000)	3.388	2.615
Postgraduates ('000)	1.733	1.447
Research income (£m, 2003 prices)	22.125	43.431

Student numbers are expressed as full-time equivalents. The early stage at which education becomes specialised in England allows us to disaggregate these, at undergraduate level, into two broad subject categories—science and non-science. Unfortunately considerations of multicollinearity preclude further disaggregation in practice.<sup>5</sup>

The costs measure includes both current and capital (in the form of depreciation) expenditures, but excludes 'hotel' type costs. These last costs, which measure costs due to the provision of student residences and catering, vary considerably from institution to institution, but they are costs that are generally recovered directly by imposing user charges, and their level in any one institution does not necessarily reflect the level of educational provision (the core business of the institution) to any great degree.

In common with many other studies (dating back as far as Cohn et al., 1989), we use research income (both from research grants and contracts and from the funding council) as a proxy for research output. The limitations of this approach have been well rehearsed in the literature. One unfortunate characteristic of the measure is that it is not possible to disaggregate research income by subject. We note that our preferred measure is very highly correlated with more output-oriented measures (such as those derived from Research Assessment Exercise scores—see, for example, <http://www.gla.ac.uk/rae/ukweight2001.xls>), and we can therefore be confident that the use of our financially based measure does not bias the key results of the present paper. In common with the majority of previous empirical studies (Cohn et al., 1989; Glass et al., 1995a, 1995b; Johns, 1997; Stevens, 2005) we do not include a measure of the knowledge and skills transfer—a role which is of increasing importance in higher education and which (after the first two 'missions' of universities, namely teaching and research) is often referred to as 'third mission' work. This is a deviation from the approach in the most recent study (Johns et al., 2005) but is a necessary omission because the complexity of the statistical technique means that the estimation of the parameters is particularly demanding (see Section 3).

Descriptive statistics appear in Table 1. One thing is very clear from these: the standard deviations for all variables are high in relation to the mean. While the means reported in the table refer, in a statistical sense, to a typical institution, the notion of such a typical institution can be very

<sup>5</sup> A referee has suggested that the value added to students' earning potential could be used as a further, or alternative measure of teaching output. While appealing, such a measure would be demanding in terms of data requirements; it would require us to be able to disentangle the impact of higher education from that of both prior education and subsequent training and experience.

misleading. The higher education sector in England is one characterised by great heterogeneity. Nonetheless, the representative model of an institution that is suggested by the means in Table 1 is one that will strike many as familiar: the university has several thousand students, roughly evenly split between the 'arts' and 'sciences', and with about one in five students studying at postgraduate level. Mean costs are a little above £85 million. These vary considerably from institution to institution, depending upon the level of production of the various outputs. The precise nature of the mapping from outputs to costs is the subject matter of the next section of this paper.

### 3. Methodology and results

Cost functions in economic theory represent an envelope or boundary which describes the lowest cost at which it is possible to produce a given vector of outputs. As it is an envelope that we wish to model, it is necessary to employ frontier methods of estimation rather than the more conventional best fit technology.

The conventional approach to stochastic frontier estimation, based upon cross-section data, is due to Aigner et al. (1977). In this model, the equation

$$y_i = \alpha + \beta'x_i + v_i \pm u_i \quad (1)$$

is estimated using maximum likelihood, where  $y_i$  is the dependent variable (typically costs or output) for the  $i$ th unit of observation,  $x_i$  is a vector of explanatory variables,  $v_i$  denotes normally distributed white noise error, typically attributed to measurement error, and  $u_i$  is a second residual term that is intended to capture efficiency differences across observations. The sum of the  $v$  and  $u$  terms equates to the total regression residual,  $\varepsilon$ . The  $u$  component of the residual could in principle follow any non-normal distribution (so that it can be distinguished from  $v$ ), though for reasons of analytical convenience the half-normal is a common assumption.

A particularly appealing feature of this approach is that, following the insight of Jondrow, Lovell, Materov, and Schmidt (1982) it is possible to recover observation-specific estimates of the efficiency residual. This estimator is given by

$$E[u_i|\varepsilon_i] = \frac{\sigma\lambda\{\phi(a_i)/[1 - \Phi(a_i)] - a_i\}}{1 + \lambda^2} \quad (2)$$

where  $\sigma = (\sigma_v^2 + \sigma_u^2)^{1/2}$ ,  $\lambda = \sigma_u/\sigma_v$ ,  $a_i = \pm\varepsilon_i\lambda/\sigma$ , and  $\phi(\cdot)$  and  $\Phi(\cdot)$  are, respectively, the density and distribution of the standard normal.

When using panel data, it is appropriate to modify (1) to

$$y_{it} = \alpha_i + \beta_i'x_{it} + v_{it} \pm u_{it} \quad (3)$$

where  $v_{it} \sim N[0, \sigma_v^2]$ ,  $u_{it} = |U_{it}|$ ,  $U_{it} \sim N[0, \sigma_{ui}^2]$ , and  $v_{it}$  is independent of  $u_{it}$ . Eq. (2) is similarly modified, for the panel data case, to

$$E[u_{it}|\varepsilon_{it}] = \frac{\sigma\lambda\{\phi(a_{it})/[1 - \Phi(a_{it})] - a_{it}\}}{1 + \lambda^2} \quad (4)$$

There are various ways in which one could implement this specification; for instance it would be possible to identify

subgroups of the sample and estimate each parameter separately for each subgroup (Johnes et al., 2005). This is, in effect, the latent class estimator (Caudill, 2003). An alternative which we shall pursue in the present paper, is to model the  $\beta_i$  as random parameters. Greene (2005) summarises the problem by defining the stochastic frontier as (3) above, the inefficiency distribution as a half-normal with mean  $\mu_i = \mu_i'z_i$  and standard deviation  $\sigma_{ui} = \sigma_u \exp(\theta_i' h_i)$ , and the parameter heterogeneity is modelled as follows:

$$\left. \begin{aligned} (\alpha_i, \beta_i) &= (\bar{\alpha}, \bar{\beta}) + \Delta_{\alpha, \beta} \mathbf{q}_i + \Gamma_{\alpha, \beta} \mathbf{w}_{\alpha, \beta i} \\ \mu_i &= \bar{\mu} + \Delta_{\mu} \mathbf{q}_i + \Gamma_{\mu} \mathbf{w}_{\mu i} \\ \theta_i &= \bar{\theta} + \Delta_{\theta} \mathbf{q}_i + \Gamma_{\theta} \mathbf{w}_{\theta i} \end{aligned} \right\} \quad (5)$$

Here the random variation appears in the random parameters vector  $\mathbf{w}_{ji}$  (where  $i$  is the index of producers and  $j$  refers to either the constant, the slope parameter, or – in more general specifications of the model – the moments of the inefficiency distribution represented by  $\mu$  and  $\theta$ ); this vector is assumed to have mean vector zero and, in the case where parameters are assumed to be normally distributed, the covariance matrix equals the identity matrix.

In common with other random parameter and random effects approaches, the parameter estimates are assumed to follow an imposed distribution (in this case normal). We are happy to follow this approach because the implications of such an assumption are well understood. It is worth noting, however, that if the true distribution of parameters differs substantially from the normal then the estimates provided by the method will be in error. In the present case there are reasons to believe that the assumption of normality is not inappropriate—there exists a clearly identifiable ‘top 5’ of institutions (comprising Oxford, Cambridge, University College London, Imperial College London and the London School of Economics), a modest number of small and highly specialised institutions, and a larger number of medium sized institutions offering comprehensive provision (though some are more obviously research intensive than others). This being so, clearly one option would be to estimate separate (fixed parameter) cost functions for each group of institutions. We prefer to let the data speak, and proceed therefore to use the random parameter model.<sup>6</sup>

The parameters of this model cannot be estimated by traditional maximum likelihood methods because the unconditional log likelihood includes within it a term containing an unclosed integral. The obvious approach to adopt in this situation is to simulate the likelihood using Monte Carlo methods. Convergence to the solution of the problem therefore entails selection of numerous random draws of parameters, and so this is inevitably a computationally intensive exercise. Speed of solution can be reduced by employing Halton (1960) sequences of quasi-random draws. Such sequences have properties that resemble ran-

dom series of numbers (and so can be used for simulation) but are in fact non-random and designed to facilitate rapid convergence in numerical integration problems. In the present case we have employed 100 Halton sequences; this is equivalent to the use of almost 1000 random simulations and is therefore in line with normal practice in Monte Carlo simulations. The simulated log likelihood function that must be maximised is

$$\log L_s = \sum_{i=1}^N \frac{1}{R} \sum_{r=1}^R \left\{ \sum_{t=1}^T \ln \Phi \left\{ \frac{\mu_{it} / (\sigma_{uir} / \sigma_v) \pm (y_{it} - \alpha_{it} - \beta_{it}' \mathbf{x}_{it}) (\sigma_{uir} / \sigma_v)}{\sqrt{\sigma_{uir}^2 + \sigma_v^2}} \right\} \right. \\ \left. - \frac{1}{2} \left\{ \frac{\mu_i \pm (y_{it} - \alpha_{it} - \beta_{it}' \mathbf{x}_{it})}{\sqrt{\sigma_{uir}^2 + \sigma_v^2}} \right\}^2 + \ln \frac{1}{\sqrt{2\pi}} - \ln \Phi \left( \frac{\mu_i}{\sigma_{uir}} \right) - \ln \sqrt{\sigma_{uir}^2 + \sigma_v^2} \right\} \quad (6)$$

The model is estimated using Limdep.

It is straightforward to observe that the traditional random effects model is a special case of the random parameters model; to be specific, the former is the case of the latter where only one parameter, namely the constant term, is allowed to vary across observations. In the results reported below, we report the random effects case as a point of comparison.

The recent literature on costs in higher education institutions is firmly built on the foundations provided in the literature on multiproduct cost functions. This literature, which developed from the investigation of contestable markets, has highlighted the difficulty of choosing a cost function that makes sense in a multiproduct context. Baumol et al. (1982) propose three possible functional forms: the CES, the quadratic, and the hybrid translog. Problems attach to the first of these (Johnes, 2004), and the last is demanding both in terms of data and its highly nonlinear specification. We therefore restrict our analysis in the present paper to the quadratic cost function.

The results of four estimates of this cost function appear in Table 2. The first two columns report ‘best fit’ estimates (that is, they are not based on a frontier analysis). Model 1 is a standard random effects model where the constant is allowed to vary across institutions following a normal distribution. Model 2 is a random parameters model where both the constant and the coefficient on the full-time equivalent number of science undergraduates are allowed to vary, each following a normal distribution. Extensive experimentation, not reported here for reasons of space, has shown that, apart from the constant, it is only the coefficient on the linear term in science undergraduates that consistently exhibits significant variation across institutions. Models 3 and 4 are frontier counterparts to models 1 and 2, respectively.

Given the presence of quadratic and interaction terms in our preferred specification, the results in Table 2 are not straightforward to interpret. So we move quickly on to discuss some more intuitive results that emerge from our analysis. For reasons of space, a detailed analysis of results for each institution is not provided here; these results may, however, be accessed at [http://www.lancs.ac.uk/people/ecagj/Table\\_of\\_Efficiencies.pdf](http://www.lancs.ac.uk/people/ecagj/Table_of_Efficiencies.pdf).

<sup>6</sup> A referee has suggested that we should augment the present analysis with a fixed effects approach. We have done so in work not reported here, but with limited success. A fixed effects analogue of the full random parameters model is costly in terms of degrees of freedom. In comparing a fixed effects with a random effects specification (where only the intercept term is allowed to vary across institutions) a Hausman test decisively found in favour of the random effects model.

**Table 2**  
Regression results

Variable	Gaussian		Frontier	
	Model 1: RE	Model 2: RPM	Model 3: RE	Model 4: RPM
constant	2.106 (53.84) <sup>a</sup>	1.083 (27.72)	−0.328 (4.41)	−0.328 (4.44)
ug sci	0.372 (9.25)	0.925 (22.80)	0.898 (12.67)	0.898 (12.90)
ug non-sci	0.280 (9.34)	0.014 (0.45)	0.211 (4.06)	0.211 (4.05)
pg	1.125 (15.22)	1.538 (20.29)	1.186 (8.83)	1.186 (8.85)
research	0.088 (28.11)	0.083 (25.96)	0.089 (16.40)	0.089 (16.46)
(ug sci) <sup>2</sup>	0.075 (8.58)	0.030 (3.44)	0.005 (0.31)	0.005 (0.31)
(ug non-sci) <sup>2</sup>	−0.014 (2.33)	0.004 (0.60)	0.023 (2.18)	0.023 (2.31)
pg <sup>2</sup>	−0.107 (4.14)	−0.161 (6.03)	−0.133 (2.53)	−0.133 (2.55)
research <sup>2</sup>	−0.0002 (15.35)	−0.0002 (14.96)	−0.0002 (7.46)	0.0004 (13.96)
ugsci × ugnonsci	−0.017 (1.46)	−0.029 (2.54)	−0.004 (0.18)	−0.004 (0.18)
ugsci × pg	−0.281 (13.54)	−0.271 (13.14)	−0.165 (4.65)	−0.165 (4.82)
ugsci × research	0.006 (10.34)	0.007 (11.56)	0.002 (2.37)	0.002 (2.30)
ugnonsci × pg	0.210 (12.14)	0.214 (12.13)	0.034 (0.99)	0.034 (1.02)
ugnonsci × res	−0.003 (5.82)	−0.002 (4.94)	−0.002 (1.82)	−0.002 (1.95)
pg × research	0.019 (16.46)	0.018 (15.14)	0.021 (8.13)	0.021 (8.56)
Random parameters <sup>b</sup>				
constant	1.867 (81.81)	1.473 (80.45)	6.700 (39.33)	6.700 (40.22)
ug science		0.030 (6.24)		1.900 (39.55)
$\sigma$	0.475 (50.47)	0.479 (50.17)	1.900 (31.93)	1.900 (31.74)
$\lambda$			6.700 (7.48)	6.700 (7.57)
log likelihood	−457.177	−432.66	−673.58	−710.58

<sup>a</sup> *t*-statistics in parentheses.<sup>b</sup> Coefficients reported here are estimates of standard deviation of normal distribution of random parameters.**Table 3**  
Summary of random parameters and efficiencies for model 4

Type of institution	Number of observations	Intercept shift	Slope shift	Efficiency
Top 5	5	3.330	1.949	0.942
Civics	6	−1.147	1.322	0.919
ExCAT & Greenfield	15	−0.150	0.983	0.844
Other pre-1992 universities	24	0.356	1.008	0.712
Post-1992 universities	33	0.215	0.921	0.859
Colleges of higher education	23	−0.661	0.794	0.499
Total	106	0.075	0.993	0.753

In Table 3, we summarise these results by reporting (unweighted) averages of some measures of interest for some groups of institutions, based on the model 4 estimates of Table 2. These are: the top 5 (identified earlier); the ‘civics’ (universities based in large cities, typically gaining their charters in the late 19th and early 20th centuries); former Colleges of Advanced Technology and other, so-called ‘Greenfield’ universities that were set up in the 1960s; other institutions that had university status before 1992; institutions that were awarded university status in 1992 (when the binary divide between polytechnics and university was abolished); and finally Colleges of Higher Education.<sup>7</sup>

The results in Table 3 suggest that there are some systematic differences across types of institution. The top 5 tend to have high fixed and (for the provision of science undergraduates) variable costs. Recall that three of these institutions are located in London and so face high prop-

erty prices. Variable costs are particularly high in Oxford and Cambridge, this being likely due to the operation of the tutorial system. The civic universities tend to have relatively low fixed costs, but high variable costs. No obvious explanation for this pattern is evident. The remaining groups of institutions do not exhibit any strong patterns in the parameters of the cost equation.

The measure of technical efficiency is highest for the top 5 and the civics, and lowest for the Colleges of Higher Education. Former Colleges of Advanced Technology, Greenfield universities, and post-1992 universities have, on average, efficiencies that, while not quite as high as those of the top 5 or the civics, are higher than those of the remaining pre-1992 institutions.

It is worth emphasising a caveat concerning the interpretation of the efficiency scores derived from these models 3 and 4, which are calculated on the basis of institution-specific parameters (the constant for model 3, and the constant and coefficient on science undergraduates for model 4). Allowing some parameters to vary by institution brings the technique closer to DEA, and a well-known drawback of DEA is that units can be seen to be efficient simply because they are different from others in the data set. Thus the apparent cost efficiency of the top 5, which are all

<sup>7</sup> The Colleges of Higher Education, sometimes referred to as Standing Conference of Principals (SCOP) colleges, are institutions that do not have university status. Some of these have their own degree awarding powers, while others offer degrees validated by other (university level) institutions. In the period since the data window to which this paper refers, many of these colleges have gained university status.

**Table 4**  
Average incremental costs

	Model 2			Model 4		
	At 100% mean output	At 80% mean output	At 120% mean output	At 100% mean output	At 80% mean output	At 120% mean output
Undergraduate science	5,516	6,262	4,770	6,452	6,958	5,946
Undergraduate non-science	2,869	2,323	3,416	3,126	2,923	3,329
Postgraduate	16,215	16,049	16,382	10,527	10,794	10,261

**Table 5**  
Economies of scale and scope

	Model 2			Model 4		
	At 100% mean output	At 80% mean output	At 120% mean output	At 100% mean output	At 80% mean output	At 120% mean output
Product-specific returns to scale						
Undergraduate science	0.87	0.90	0.82	0.98	0.98	0.97
Undergraduate non-science	0.96	0.96	0.96	0.79	0.81	0.77
Postgraduate	1.22	1.17	1.28	1.30	1.22	1.40
Research	1.04	1.04	1.05	1.08	1.07	1.09
Ray returns to scale	1.10	1.15	1.07	0.97	0.96	0.98
Returns to scope	0.30	0.40	0.23	−0.17	−0.20	−0.15

relatively high-cost institutions, is questionable. It would follow that some of the seemingly less efficient institutions could potentially become more efficient by attempting to emulate, for example, Oxford and Cambridge. Yet encouraging institutions to become more like Oxford and Cambridge is not a practical or desirable policy for achieving cost efficiency either in the individual institutions or in the sector as a whole.

A further caveat, alluded to earlier, concerns the assumptions made about the distributions of random parameters and efficiencies. Given the imposition of a normal distribution of parameters, it is perhaps not surprising to find that a small number of universities (the top 5) have estimated parameters representing fixed and variable costs that are very high in relation to other institutions. If half of our sample of institutions had similar characteristics to Oxford and Cambridge, then the method may not be capable of identifying high costs for such a large fraction of the sample. But this is not the case, and, as argued earlier, we deem the assumption of a normal distribution to be a reasonable one to make in the present context.

A final caveat which we should make explicit at this stage concerns the length of the panel. Three years is a short time, and so much of the variation used to perform the statistical analysis comes from the cross-section dimension of the panel. The optimum length of a panel is, of course, a judgement call—a longer panel might lead to concern that cost structures within each institution are changing within the time frame.

Much of the interest in studies of the cost structures of multiproduct institutions comes from statistics on average incremental costs associated with each output, and from statistics on economies of scale and scope. Standard measures of these were defined by Baumol et al. (1982) and have been used in numerous studies – including Cohn et al. (1989), Johnes (1997) and Johnes et al. (2005) – since. These now being standard and well understood definitions, we do not define them here, but proceed to report the vari-

ous statistics that emerge from analysis of the two random parameter models—model 2 which follows the ‘best fit’ approach and model 4 which follows the frontier approach.

Average incremental costs are shown in Table 4. These are reported, for each of the models, for a representative institution (namely one producing the mean level of each of the outputs), and also for institutions that produce 80% and 120%, respectively of the mean of each output type.<sup>8</sup> The results indicate that science undergraduates cost between twice and three times as much to produce as do non-science undergraduates, and that postgraduate education is markedly more costly than undergraduate education. It is noticeable, however, that the frontier model estimates the average incremental costs associated with postgraduate education to be markedly lower than is the case with the ‘best fit’ model.<sup>9</sup>

The results shown in Table 5 indicate that product-specific returns to scale are exhausted for undergraduates in institutions close to the representative size. Economies of scale remain unexhausted in the context of postgraduate education and research, however. These results are robust with respect to choice of estimation method. They accord with the results presented in Johnes et al. (2005), Johnes (1997), using data for an earlier period and for a smaller sample of institutions, finds that product-specific economies of scale are exhausted for science undergraduates, but not for arts undergraduates.

<sup>8</sup> The results in Tables 4 and 5 are based on the average value of the coefficients for the random parameters.

<sup>9</sup> One interpretation of this is that the ‘best fit’ model refers to actual expenditures rather than to the costs that need to be spent by an efficient institution. Bowen (1980) has argued that ‘each institution raises all the money it can’ and ‘each institution spends all it raises’. If an institution can raise funds by hiking tuition for one output type – say postgraduates – then estimation of the equation by means of a ‘best fit’ method will tend to indicate that more postgraduates imply more expenditure. The frontier model does not suffer from this problem, since any expenditure that is above the cost frontier is attributed to inefficiency.

Findings on ray returns to scale and on returns to scope are sensitive to the choice of estimation methodology. Using a ‘best fit’ method, ray economies of scale appear to be unexhausted, this being in large measure due to the fact that returns to scope are positive. However, using a frontier method, these results are reversed. This finding has clear implications for the further expansion of higher education in the UK. If current efficiency levels are taken as given, any further expansion of higher education should (in order to minimise global costs) be effected within the existing institutions. If, however, efficiency could be increased, overheads would fall and hence the opening of new institutions would become a viable option.

#### 4. Conclusions

Earlier studies which have estimated cost functions for institutions of higher education have failed to recognise that, owing to unobserved heterogeneity, each institution likely faces a different cost function. In this paper, we use methods that have recently become available to estimate frontier cost functions for higher education institutions within the context of a random parameter model. This brings the analysis somewhat closer to the spirit of non-parametric techniques such as DEA (and therefore has some of its drawbacks, such as its sensitivity to the presence of outliers), and allows questions to be answered about the distinction between inefficiency and idiosyncratic cost technologies. By allowing parameters to vary across institutions, cost functions for institutions that are obviously quite different from one another can be estimated within a single, unified framework, obviating the need for separate equations to be estimated for exogenously determined groups of institutions.

Our findings on returns to scale and scope, and on average incremental costs have much in common with the received literature. Findings that are new primarily concern the decomposition of cost differentials into components due to differences in cost technology, on the one hand, and efficiency, on the other. So, for example, while Izadi, Johnes, Oskrochi, and Crouchley (2002) comment on the London Business School (which in that study had a low measured efficiency score) as an idiosyncratic case, it is clear from the present analysis that the higher than expected costs of that institution are due in part to an unusual cost technology, and in part to efficiency issues.

Simple frontier models exist that simultaneously determine efficiency scores and explain them by reference to a vector of (environmental) variables. Such models have not

yet been extended so that they can be used in a random parameter context. That would be an obvious development of the present work that must be left to the future.

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