Corporate Finance and the (In)efficient Exercise of Real Options

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Abstract
This paper considers real options within a continuous-time corporate finance context. We analyze whether these real options are exercised efficiently, and what the underlying sources of inefficiency are. In particular we consider the role of incomplete information, competition, search costs and financing constraints on investment decisions. We also analyze the stockholder-bondholder and the manager-stockholder agency problems, and their effect on a firm's investment and closure policies.

Keywords: real options, product market competition, costly search, financing constraints, agency problem

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1 Introduction

One of the key features of real option valuation models is the determination of the optimal exercise of those real options. The value of a real option essentially depends on the time when the option is exercised. Investors choose the exercise strategy to maximize the value of their real options. In this article we analyze the conditions under which the exercise of real options happens at the efficient time (from a global optimizer's viewpoint), inefficiently early, or inefficiently late. The analysis allows us to make a direct parallel with the long standing literature on the efficiency of investment decisions. In this literature inefficiency is typically expressed in terms of overinvestment or underinvestment: firms invest (or produce) too much or too little compared to what is socially desirable. Since the exercise of real options is essentially a timing problem, the analogue is whether firms invest too early or too late compared to the first-best investment time. Take the case of market entry, for example. If entry happens under, say, product price uncertainty then the investment happens inefficiently early (late) if the market entry happens at a price level that is below (above) the globally optimal price level. The value that is lost (compared to the first-best outcome) can often be interpreted as the agency cost.

One may wonder how deviations from the first-best outcome can arise. We discuss this at great length in this article, and at this stage only provide a couple of examples to illustrate the paper's key issue. Underinvestment happens, for example, when one party is making an investment under the form of (costly) effort or capital, but is not getting the full benefit of the investment made because part of the benefits have to be shared with a second party. A well known example is the Myers (1977) underinvestment problem where equityholders of a levered firm refuse to contribute capital to finance positive NPV projects when the firm is close to bankruptcy. The reason is that if the firm ends up going bankrupt shareholders do not (fully) benefit from the capital they injected as they are residual claimants in liquidation (with bondholders having a higher priority). Overinvestment occurs, for example, when one party through his or her actions is imposing negative externalities onto another party. The Jensen and
Meckling (1976) asset substitution problem is a typical example of this. Equityholders of a levered firm can potentially extract value from bondholders by increasing investment risk (i.e., increase the riskiness of the firm's assets) after the debt is in place. The limited liability feature of equity causes it to have (call) option like features. Increasing the risk of the firm's assets raises the value of the equity for the same reason that the value of a stock option increases with the volatility of the underlying stock. The above two examples have been the building blocks of many contingent claims models that model corporate bankruptcy and the associated agency costs of debt (e.g., Mello and Parsons (1992), Leland (1994), Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Leland (1998), Lambrecht (2001) and Morellec (2004)).

The objective of this paper is not to give an exhaustive review of all real options papers that in one way or another have an agency component to them. Rather we identify a few prominent corporate investment problems from corporate finance and industrial economics and discuss how real options theory can enhance our understanding of those problems by providing us with new insights on the timing of corporate investment under uncertainty.

The structure of the paper is as follows. Section 1 (“The timing of market entry and the paradox of competition”) considers the effect of competition on the timing of market entry. We consider the cases of monopolistic, duopolistic and perfect competition. Our comparisons lead to some findings that at first sight seem surprising or paradoxical, but that can be explained on the basis of simple economic principles. Section 2 (“Investment, costly search and financing constraints”) presents two situations in which investment policy is adversely affected by some form of market imperfection such as search costs and financing constraints. First, investors have to search for investment opportunities. When an investment opportunity arises investors decide whether to accept the project given that they do not know when or whether a better investment opportunity will come along in future. We show that there is a close parallel between the costly search investment problem and the entry decision under preemption with incomplete information about the competitor. A similar trade-off exists when investors are financially constrained: the benefit of waiting for better investment opportunities is reduced by a positive
probability of having insufficient resources to finance the project in the future. Section 3 (“The shareholder-bondholder conflict of interest”) discusses the agency problem between a firm's shareholders and bondholders, and how this affects the exercise of real options. In particular we discuss the Myers (1977) debt overhang problem and the Jensen and Meckling (1976) asset substitution problem within a real options context. Section 4 (“The manager-shareholder conflict of interest”) discusses sources and consequences of the agency problem between managers and the shareholders they are meant to represent. We discuss the problem both within the context of complete and asymmetric information. Section 5 (“The exercise of collectively held options”) considers real options that are held by two parties. Both parties therefore have to agree to exercise the real option at the same time and how to share the proceeds of this option. We focus on the example of mergers and analyze the implications for the timing and terms of mergers.

2 The timing of market entry and the paradox of competition

In this section the effect of competition of a firm's decision to enter a market is analyzed. We contrast the single firm case with the two-firm case and the case of perfect competition. We conclude by discussing the role of incomplete information on the strategic entry decision.

Consider a simple framework where a single firm can enter a market by paying a sunk cost $K$. Once the firm has entered it can sell in perpetuity a product of which the output price is given by $p_t = x_t D(Q)$. $x_t$ follows a geometric Brownian motion with drift $\mu$ and volatility parameter $\sigma$, and $D(Q)$ is the non-stochastic part of the industry's inverse demand curve. The present value at time $t$ of all expected future profits is therefore $\frac{Q^* D(Q^*) x_t}{r - \mu} = \frac{\pi x_t}{r - \mu}$, where $Q^*$ is the monopolist's optimal output level and where we assumed for simplicity that the firm's production cost is zero.
Real option theory says that this monopolistic firm will enter at some threshold \( \bar{x} \) which exceeds the Marshallian breakeven threshold \( \bar{x}_m \), where \( \bar{x}_m = \frac{K(r - \mu)}{\pi} \). The real option trigger strikes an optimal balance between the benefit of waiting (due to the irreversible nature of entry) and the cost of waiting (in terms of the profits foregone), and is given by \( \bar{x} = \frac{\beta K(r - \mu)}{(\beta - 1)\pi} \).

Suppose we now introduce a second, identical firm within our model and assume that the first firm to enter the market acquires the whole market in perpetuity. Since the after entry the market cannot be contested the entrant becomes a de facto monopolist and acquires the same profit stream as in the above single firm case. However, the entry threshold will now be radically different, as one can show that the equilibrium entry threshold is now given by the Marshallian breakeven point, \( \bar{x}_m \). Indeed if firm 1 were to adopt a higher threshold \( \bar{x} (> \bar{x}_m) \) then firm 2 would have an incentive to "epsilon preempt" by entering at \( \bar{x} - \varepsilon \). This in turn would give firm 1 an incentive to act at \( \bar{x} - 2\varepsilon \) and so. The only credible entry trigger is therefore the breakeven trigger. Competition between two identical firms with an absolute first mover's advantage (i.e., winner takes all) is therefore sufficient to compete away all option value of waiting. This outcome is equivalent to a situation of overinvestment, i.e., firms invest inefficiently early. The result is completely in line with the industrial organization literature on R&D and patent races, for example, which argues that competition typically leads to overinvestment in R&D (see, e.g., Dasgupta and Stiglitz (1980) and Fudenberg and Tirole (1985)).

Let us contrast now the above two cases with the polar case of perfect competition. This case is analyzed in Leahy (1993), and we follow his framework for our discussion. We assume there is a large number of competitive firms. Each firm can enter the market by paying the sunk cost \( K \). Once this investment is made it yields a flow of one unit of output forever. We assume that one unit of output is very small compared to the total industry output \( Q \) so that each firm
considers itself as an infinitesimal price taker. Leahy (1993) shows that there exists a threshold \( \bar{p} \) at which firms enter the market. Moreover, perfect competition ensures that the output price never exceeds \( \bar{p} \). How is this possible given that \( p_t = x_t D(Q) \) and \( x_t \) can rise arbitrarily high?

Whenever the stochastic variable \( x_t \) rises and causes the output price \( p_t \) to hit the threshold \( \bar{p} \), new firms enter increasing the supply \( Q \). Perfect competition will ensure that the supply is exactly high enough to prevent the output price from exceeding the threshold \( \bar{p} \). It follows therefore that the output price is characterized by a geometric Brownian motion with a reflecting barrier at \( \bar{p} \). More importantly, Leahy (1993) shows that the entry threshold \( \bar{p} \) is exactly the one that would be adopted by a unit-sized monopolist firm facing the same demand process, i.e. 

\[
\bar{p} = \frac{\beta K}{(\beta - 1)(r - \mu)}
\]

(hence the subtitle of his paper "the optimality of myopic behavior", referring to the fact that each firm can pretend as if the other firms are not there). At first sight, this result may seem paradoxical and difficult to square with our previous result in a duopoly setting where competition between two firms is sufficient to obtain the Marshallian breakeven threshold. The discrepancy results from the fact that in our two firm model there was an absolute first mover's advantage in the sense that only one firm is allowed to enter, whereas in the competitive equilibrium this is not the case: firms can enter whenever they want, and all firms in the market have the same market value. Let us explain this in more detail. Consider first the case of perfect competition. Competition here has two effects. First, it limits the upside potential of the price process (which is now reflected at \( \bar{p} \)). This, in turn, causes the present value of a unit sized firm to be less than \( \frac{p_t}{r - \mu} \), which would be its value if the price process were not truncated. To make up for this cap on the product price firms would therefore tend to increase the price at which firms enter the market. Moreover, for entry to be viable, the firm value upon entry needs to cover the cost of entry \( K \), which requires the entry price to be situated above the Marshallian investment trigger. Competition, however, also has a second
effect. While a single firm can fully exploit its option value of waiting, perfect competition creates a downward pressure on the entry price and completely dissipates this option value of waiting to invest. It happens that both effects (the price cap and the dissipation of all option value) exactly offset each other such that under perfect competition firms invest at the non-strategic single firm investment threshold. We do not obtain this threshold in the two-firm winner-take-all scenario. In that scenario competition only affects the pre-entry stage, but not the product market competition itself (since only one firm is allowed to operate in the market). The latter implies that the price process will not be truncated and therefore the present value of a unit sized firm upon entry equals \( \frac{\mu - \rho}{r - \mu} \). Competition in the pre-entry stage then merely destroys all the option value of waiting to invest, so that firms invest at the Marshallian threshold. Ex-post (once the entry cost is sunk) the value of the unit sized monopolist that arises from the duopoly game will therefore be higher than the value of a unit sized firm in a perfect competitive market.

The above analysis of first mover's advantage and perfect competition leads to the conclusion that competition erodes most, if not all, option value of waiting. In practice, however, we rarely observe this kind of extreme behavior. Investors of delay beyond the breakeven trigger, even when there are first mover's advantages. Lambrecht and Perraudin (2003) argue that one of the reasons for this is the fact that investors make their decisions under incomplete information about their competitors. Incomplete information prevents investors from ‘epsilon’ preempting their competitors as can happen under complete information, and consequently more option value of waiting is preserved, restoring to some extent efficiency. Under incomplete information firms strike an optimal balance between the benefit of waiting and the cost of being preempted by a competitor. Lambrecht and Perraudin (2003) consider the above described duopoly case, but where firms have incomplete information about the entry cost of their opponent. In particular the entry cost is drawn from some distribution \( G(K) \). They show that this distribution for the competitor's entry cost implies in equilibrium a distribution
\( F(\bar{x}) \) for the rival's entry threshold. A firm's entry threshold is then given by:

\[
x = \frac{(\beta_1 + h(\bar{x})(r - \mu)K)}{\beta_1 - 1 + h(\bar{x})}
\]

(1)

where \( h(x) \) reflects the hazard rate of being preempted, i.e.,

\[
h(x) = \frac{x F'(x)}{1 - F(x)}.
\]

One can easily see that if there is no fear of preemption \( h(x) \equiv 0 \), then investment happens at the usual non-strategic real options threshold. As the competitive threat increases \( h(x) \to \infty \), the investment threshold converges to the Marshallian breakeven threshold, and more and more option value of waiting is destroyed, which implies an increasing degree of overinvestment.

3 Investment, costly search and financing constraints

In previous section we considered the case where firms have an option to invest, but this investment opportunity can at any time disappear (or be reduced in value) because of the arrival of a competitor. Investors therefore have to make a tradeoff between the value of waiting and the cost of being preempted.

One could also imagine the scenario where some form of market imperfections exists and investment opportunities can be exercised only if external circumstances are favorable. Those circumstances may occur in a random, unpredictable fashion, and remain beyond the control of the firm. When such a “window of opportunity” arises investors often have to decide there and then whether to accept the project (“the bird in the hand”) or whether to reject it and wait for a better investment opportunity that may (or may not) arrive in future (“the bird in the bush”). In this section we discuss the cases of costly search and financing constraints, which exemplify such imperfections.

3.1 Investment with costly search
This investment problem with costly search was modelled by Williams (1995) in the context of the real estate market. Williams assumes that in order to develop a property, its owner needs to hold an offer from a developer. Offers arrive with a Poisson arrival rate $\delta$ and expire instantaneously if not accepted. Therefore, $\delta$ tending to zero implies that the offers are so scarce that finding a developer is very unlikely. Conversely, $\delta$ tending to infinity means that offers are abundant and the owner can act as if there were no frictions. The cost of developing the property is $K$ and the value of the developed property equals $V_t = \frac{x_t}{r - \mu}$. In general, developing the property by a developer is costly and it is assumed that the surplus from development is divided between the owner and the developer according to the owner's bargaining power parameter $\omega \in (0, 1)$. In other words, the owner receives $\omega(V_t - K)$ and the developer is compensated with the remaining fraction of the project's NPV. The investment decision with a costly search reflects the fact that there is no guarantee for the owner that the development will take place at a desired moment. The act of development ultimately depends on holding an offer at a given instant. Consequently, the decision to invest is in fact a decision to be ready to accept an offer upon its arrival.\(^4\)

The investment trigger with costly search, $x_{cs}$, is determined by comparing the value of the undeveloped property, which equals the option to enter the region of optimal development, with the value of the developed property, $V_t - K$. The value of the undeveloped property is determined by applying the smooth-pasting condition at $x_{cs}$. In the region below $x_{cs}$, the property value equals the value of the option to enter the region of optimal development. The value of the property in the region above $x_{cs}$ reflects the fact that investment occurs only if the offer is available. Consequently, it equals the net present value of the project accruing to the owner $\omega(V_t - K)$ multiplied by the arrival rate $\delta$ and adjusted by the components reflecting a positive probability of leaving the optimal development region before the offer is received.
One can show that the optimal investment threshold with costly search $x_{cs}$, equals

$$x_{cs} = \frac{\left(\beta_1 - \beta_2^* \frac{r}{r + \delta \omega}\right) K}{\beta_1 - \frac{\delta \omega}{r + \delta \omega - \mu} - \beta_2^* \frac{r - \mu}{r + \delta \omega - \mu}},$$

(2)

where $\beta_2^*$ is equal to $\beta_2$ (defined in the appendix) with $r$ replaced by $r + \delta \omega$. The optimal threshold $x_{cs}$ can be shown to be smaller than the standard investment threshold without costly search, $x$ (cf. (13)). In the situation when offers arrive more frequently, i.e., when the risk of being left with an undeveloped property is low, the owner can afford waiting longer and the optimal threshold $x_{cs}$ is closer to $x$. In the limit, i.e., when $\delta$ tends to infinity, the optimal threshold $x_{cs}$ converges to $x$. Conversely, if the probability of receiving an offer is relatively small ($\delta$ close to zero), the owner is willing to accept an incoming offer at any time as long as the option to invest is in-the-money. In the limit, i.e., when $\delta$ tends to zero, the owner applies the NPV rule.

The optimal investment rule with costly search closely resembles the result of Baldwin and Meyer (1979), who present a model of a firm searching for investment opportunities. Those opportunities arrive and expire according to Poisson processes. Baldwin and Meyer show that the firm's reservation value (interpretable as $x_{cs}$ in Williams' setting) increases in the project's arrival rate.

Note also the similarity between the investment threshold when there is a hazard of preemption (equation (1)) and the investment threshold under costly search (equation (2)). Just as a higher threat of preemption speeds up investment, a lower probability of receiving investment opportunities in future also makes investors eager to invest sooner.\(^5\)

### 3.2 Investment with financing constraints

Another source of market imperfection is restricted access to external financing. For instance,
asymmetric information (see Myers and Majluf (1984), and Stiglitz and Weiss (1981)) can lead to the rejection of good investment opportunities because external financing may be deemed overly expensive for the management (whose inside information is superior to that of outside investors). Boyle and Guthrie (2003) use the real options framework to analyze the investment decision of a firm which is financially constrained.

In general, there are two effects of financing constraints on the investment threshold, $x_{fc}$. The first (obvious) effect is that the firm may not have sufficient resources to finance the investment even if the current value of the project exceeds the breakeven value $\bar{V} = \frac{\bar{x}}{r - \mu}$. In such a situation, investment is triggered by an increase in cash stock of the firm beyond the level of $K$. This observation is consistent with Fazzari, Hubbard and Petersen (1988), who provide an evidence that investment of financially constrained firms is sensitive to changes in their cash balances.

Second, when the firm's current resources are sufficient for undertaking the project (i.e., when they exceed $K$), the firm may find it optimal to invest even if the value of the project is lower than $\bar{V}$. Boyle and Guthrie (2003) assume that the firm can only make an investment if the level of accumulated cash, $S_t$, combined with the resale value of firm's assets, $A$, and with the collateralizable fraction of the project's value, $\alpha V_t$, $(0 \leq \alpha < 1)$, exceeds investment cost $K$. Cash balance $S_t$ of the firm has two sources: the riskless interest on the existing cash balance and stochastic proceeds (positive or negative) from the firm's operations. Consequently, the problem analyzed by Boyle and Guthrie (2003) corresponds to the basic real option problem with the restriction that the investment can be made if and only if $S_t + A + \alpha V_t \geq K$.

If the firm has sufficient liquidity to finance the project mainly due to a high level of $S_t$, the investment threshold with financing constraints, $x_{fc}$, is lower than the unrestricted threshold $\bar{x}$. The result is due to the fact that the value of waiting is reduced by the possibility of future
cash shortages. This "bird in the hand" strategy of a firm facing financing constraints therefore closely resembles the investment policy of a firm with costly search. In both cases the window for exercising the investment opportunity may disappear and the trade-off between exercising immediately and waiting for a higher realization of the project's value arises.

Boyle and Guthrie (2003) show that the investment threshold under financing constraints $x_{fc}$ increases with $S_i$ and tends asymptotically to the unconstrained trigger $x$ for $S_i \to \infty$. Consequently, financially unconstrained firms overinvest (i.e., invest too early) and the magnitude of overinvestment is positively related to the degree of financing constraints (measured by the level of $S_i$). Moreover, concavity of $x_{fc}$ in $S_i$ implies that the value of the waiting option is more sensitive to the cash balance for relatively more cash constrained firms. Since the cash flow sensitivity of investment is inversely related to the cash flow sensitivity of the waiting option, the result of Boyle and Guthrie (2003) is consistent with Kaplan and Zingales (1997). (Kaplan and Zingales show that relatively less financially constrained firms exhibit a higher investment-cash flow sensitivity.)

If the firm's cash balance $S_i$ is sufficiently low, the risk of future cash shortages outweighs the benefit of waiting. In that case the financing constraint is binding and the investment threshold of the firm equals

$$x_{fc} = \frac{K - S_i - A}{\alpha (r - \mu)}$$

This means that for a low cash balance, the firm finds it optimal to kill the entire option value of waiting and to invest as soon as the value of the project reaches the level at which its cash balance $S_i$ and collateral $\alpha V_t + A$ are sufficient to cover $K$.

4 The shareholder-bondholder conflict of interest

We focus primarily on the conflict of interest that arises in the run-down to bankruptcy. We
discuss a number of key factors that influence bankruptcy and their role for under or overinvestment.

4.1 The role of bankruptcy costs

We interpret bankruptcy costs primarily in terms of direct bankruptcy costs that are incurred once the firm is effectively liquidated (in contrast to indirect bankruptcy costs that are incurred in the run-down to bankruptcy). Direct bankruptcy costs are primarily borne by senior creditors, because equityholders, as residual claimants, receive little or nothing of the liquidation proceeds. This again creates a conflict of interest between stockholders and bondholders. When making their decision to close the firm and to file for bankruptcy, stockholders do not take into account the bankruptcy costs borne by the creditors. Closure therefore happens inefficiently early compared to the time that would be chosen by a global optimizer who fully internalizes the costs of bankruptcy.

One possible way to mitigate those inefficiencies is by renegotiating the debt contract. Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997), Mella-Barral (1999) and Lambrecht (2001), among others, show that it may be in the creditors collective interest to make concessions in the run-down to liquidation. Those concessions can take the form of, for example, debt exchange offers (whereby the old debt contract is exchanged for a new debt contract with more lenient terms such as longer maturity or lower interest repayments) or temporary debt holidays. The creditors rationale for making concessions is that it postpones bankruptcy (and hence bankruptcy costs) and improves the chances of the firm recovering (and subsequently paying off the loan). Obviously, the higher the costs of bankruptcy, the larger the concessions creditors may be prepared to make.

4.2 The role of the firm’s liquidation value
It is well known that the value of a firm as a going concern varies over time. One simple reason, for example, is that the firm's value depends on uncertain operating profits. That also the value of a firm in liquidation may vary over time is less well recognized. For example, even though a machine may retain the same level of productivity over its life time, its market value often depreciates exponentially over time. Another commonly observed regularity is that a firm's assets in liquidation are often worth more in times when the industry is doing well than in economic downturns. One explanation is that in recessions many firms may be liquidating assets, and this at a time when there is very little demand for them from other firms.

It is then easy to see that if the liquidation value of assets falls at a fast rate in the rundown to closure this may lead to inefficiently late closure. Again, since equityholders do not benefit from the liquidation proceeds they do not internalize the asset's depreciation rate nor the asset's opportunity cost of capital when making their closure decision. One possible way to overcome this inefficiency is to allow for deviations in absolute priority (see, e.g., Mella-Barral (1999)). Empirical studies (e.g., Franks and Torous (1989)) have documented deviations from the absolute priority rule and attributed this partly to the debtors' ability to defer liquidation by entering the firm into a lengthy and costly reorganization procedure such as Chapter 11.

4.3 The debt overhang problem

Another possible source of conflict of interest between stockholders and bondholders is the so-called debt overhang problem, as analyzed in the Myers (1977) seminal paper. This problem arises when firms have debt financing and are in financial distress. The problem essentially consists of equityholders refusing to contribute capital to investments or operations that have a positive net present value (from the firm's viewpoint). The reason is that too much of the value created through the equityholders' capital injections is being shared with the bondholders. For example, a dollar invested by equityholders may increase the total firm value by 1.5 dollar. Yet, if the firm is likely to default in the near future, the value of the equity may only rise by, say, 0.8
dollar, with the remaining 0.7 dollar going to the bondholders. In the extreme case where
bankruptcy is imminent, any benefits from cash injected by equityholders goes almost entirely
to the bondholders who have the first claim on the firm's assets in liquidation.

An important consequence of the debt overhang problem is that equityholders close the
firm too early (i.e., they underinvest). This causes the firm value to be below its first-best value,
resulting in agency costs. Those agency costs, in turn, cause investments in distressed firms to
happen inefficiently late.

Pawlina (2005) analyzes investment policy when equityholders have an option to
renegotiate the debt contract when the profit of the firm declines. He shows that the presence of
the renegotiation option exacerbates the underinvestment problem compared to the situation
where renegotiation is not possible (cf. Mauer and Ott (2000)). The adverse impact of the
renegotiation option on investment results from the negative externality that the exercise of the
investment opportunity imposes on the option value to renegotiate and that is taken into account
when determining the investment policy. Since the value of this externality exceeds the negative
impact of investment on the value of bankruptcy option, investment occurs later than when
future renegotiation is not allowed for.

4.4 The asset substitution problem

Asset substitution is a consequence of the objectives of equityholders being divergent from
those of creditors. Jensen and Meckling (1976) show that inefficiencies in the firm's operational
policy occurs because equityholders tend to choose strategies that are too risky from a global
(i.e., total firm value) optimizer's viewpoint. This results from the fact that the value function of
shareholders is convex, whereas the value function of creditors is concave.

The problem of the asset substitution can be formulated in a real options framework as the
decision to switch between two stochastic processes that govern the value of the firm's securities
(see, e.g., Leland (1998), Ericsson (2000) and Dangl and Lehar (2004)), i.e.:
$$dx_t = \begin{cases} 
\mu(L)x_t dt + \sigma_L x_t dw_t & \text{when low risk strategy is chosen,} 
\mu(H)x_t dt + \sigma_H x_t dw_t & \text{when high risk strategy is chosen,} 
\end{cases}$$

and where $\sigma_L < \sigma_H$. The riskiness of a given strategy is thus represented by the volatility of its corresponding stochastic process. Equityholders can choose at any point in time to costlessly switch from the low-risk to the high-risk strategy and they are allowed for an arbitrary number of switches.\(^8\) The firm's value-maximizing switching policy is either to choose the low-risk strategy for all $x_t$ or to choose the low-risk strategy for relatively high values of $x_t$ and the high-risk strategy in the neighborhood of the bankruptcy threshold. In general, equityholders switch to the high-risk (low-risk) strategy too early (too late), i.e., for too high a level of firm's profits.

A possible way to eliminate asset substitution is suggested by Bhattacharya et al. (2002) in a banking sector application. The authors assume that the value of a bank's equity includes an element of a subsidy. They determine the endogenous closure trigger imposed by the regulator so that the option value of limited liability is cancelled out by the component reflecting the loss of the subsidy.\(^9\) In other words, due to the regulator's policy, the payoff to equityholders becomes linear in $x_t$ and the problem of asset substitution no longer exists. In a related contribution, Dangl and Lehar (2004), asset substitution is associated with a conflict of interest between the bank's equityholders and the deposit insurer. Dangl and Lehar propose that the level of asset substitution can be reduced by imposing a higher closure threshold for a bank that pursues the high-risk strategy.

The magnitude of asset substitution between shareholders and creditors as measured by the ratio of the value of the firm pursuing the first- and second best risk management policies is reported to be relatively small. For example, using realistic parameter values Leland (1998) estimates that the agency costs of debt due to asset substitution amount to less than 1.5% of the firm value.

The asset substitution problem occurs not only between shareholders and creditors but
also between the firm's manager and shareholders. (The manager-shareholder conflict of interest is discussed in great detail in section 5.) Subramanian (2002) shows that the magnitude of the asset substitution problem increases if one allows for imperfect alignment of the managerial objectives with the shareholder's value. An introduction of an impatient and risk-averse manager who is not able to diversify away the firm's risk, combined with the assumption that $\mu(\sigma_H)$ exceeds $\mu(\sigma_L)$, results in a higher magnitude of the costs of asset substitution (Subramanian (2002) estimates that these costs can easily exceed 6% of the firm value). As a solution to this problem, Subramanian (2003) proposes a contract that contains a floor and a cap on the manager's payoff. The floor is aimed at preventing the manager from switching to the low-risk strategy in a period of declining profit, whereas the cap makes the manager switch to such a strategy for high realizations of earnings.

5 The manager-shareholder conflict of interest

Managers are supposed to act in the best interest of the outside equityholders. However, often their interests are not perfectly aligned, creating conflicts of interests which in turn lead to managerial decisions that are suboptimal from the shareholders' viewpoint and from a global optimizer's viewpoint. The resulting loss in value is called the managerial agency cost. This agency problem has been discussed in the economics literature for over 30 years. More recently, a number of papers have tackled the problem within a continuous-time corporate finance setting (e.g., Mæland (1999), Morellec (2004), Grenadier and Wang (2004) and Lambrecht and Myers (2007)). The papers can be split into two categories: the complete information models (Morellec (2004) and Lambrecht and Myers (2007)) and the models with asymmetric information (Mæland (1999), Grenadier and Wang (2004)). The latter category directly extends the classical principal agent problems under asymmetric information.
5.1 Complete information models

If shareholders have complete information about what's going on in the firm, one might wonder how managers can get away with decisions that are not in the shareholders' interest. Lambrecht and Myers (2007) argue that it may be costly for shareholders to take collective action. Moreover, those costs may not be borne by all shareholders in proportion to their shareholding; often small shareholders may freeride on the efforts of large shareholders exacerbating the cost of collective action for the latter. This cost creates space for managers to extract free cashflows from the company: as long as management does not take too much, outside shareholders do not find it worthwhile to intervene. While this cost of collective action allows management to extract free cash flows, it is important to stress that this does not necessarily also lead to inefficient exercise of real options. If, for example, the cost of collective action implied that management could expropriate 10% of all cashflows (including proceeds of liquidating assets) paid out to shareholders, then the management's interests would be perfectly aligned with shareholders' (management essentially receives a fixed fraction of all future cashflows and therefore has an interest to maximize the present value of all future cashflows to shareholders). In reality, however, shareholders' rights on cashflows are more difficult to enforce than their property rights on the firm's capital assets. In particular, shareholders' property rights on capital released from closing the firm or selling off assets are much easier to enforce than their rights on free cashflows generated by the firm's operations. Lambrecht and Myers (2007) show that this gives management an incentive to close or contract firms inefficiently late since management does not fully take into account the opportunity cost of the firm's stock of capital. Those agency costs cause firms to trade below their first-best value and creates a role for raiders and takeovers. Raiders and acquirers typically have a lower cost of collective action than inside shareholders and can therefore create value by acquiring and restructuring firms that are inefficiently run. Lambrecht and Myers (2007) the efficiency of four different types of acquisition (raiders, hostile takeovers, management buyouts and mergers). They show that the
first two are efficient whereas the latter two lead to inefficiently late closure of the target firm.

Morellec (2004) analyzes the effect of the manager-shareholder conflict of interest on leverage and firm value in a contingent claims model where the manager derives perks from investment. The conflict of interest arises from the assumption that the manager derives utility from both retaining control and from investing in new projects. This causes managers to overinvest. When capital structure decisions are made by the manager, empire-building desires induce entrenched managers to issue less debt than optimal. Morellec (2004) argues that the manager-shareholder conflict can explain the low debt levels observed in practice.

5.2 Incomplete information models

Another important reason why managers can undertake decisions that are not in shareholders' interest is that managers are often better informed. In this subsection we discuss a situation in which the manager has private information about the investment cost or/and can influence its level by exerting effort. In such a case two types of agency problems may occur. First, the manager may not disclose the true value of the investment cost, so adverse selection takes place. Second, they may refrain from the optimal (i.e., investment value-maximizing) level of effort, so one can talk about moral hazard. In general, adverse selection and moral hazard may occur simultaneously.\textsuperscript{10}

First, we consider a situation in which only adverse selection is present. The manager (the agent) has private information about the true investment cost, $K$. The owner of the firm (the principal) knows only its probability density function, $\phi(K)$. As in the standard optimal contracting literature, an application of the revelation principle (cf. Salanie (1997)) allows the principal to restrict to the class of contracts that induce the agent to report the truth (as no other contracts strictly dominate those based on truth-telling). Then, the principal has to ensure that truth-telling is incentive compatible for the agent for any realization of $\tilde{K}$. The principal has
two instruments that can be used to induce the agent to tell the truth: the compensation and the timing of investment (both being functions of the investment cost reported by the agent, $\hat{K}$).

Maeland (1999) shows that the optimal investment threshold equals

$$
\bar{x}_{\text{opt}} = \frac{\beta_1}{\beta_1 - 1} \left( K + \frac{\Phi(K)}{\phi(K)} \right) (r - \mu),
$$

(4)

where $\Phi(K)$ is the cumulative density function of the investment cost.\(^1\) From (4) it can be immediately seen that the investment threshold in the presence of hidden information is in general higher than the optimal investment threshold $\bar{x}$.\(^2\) The investment inefficiency occurs as long as the true investment cost is strictly greater than the lower bound of the support of $\phi(K)$, $K$. Moreover, the magnitude of inefficiency increases with $K$.

The reason of the departure from the first-best investment policy is as follows. As long as the investment cost is strictly lower from the upper bound of the support of $\phi(K)$, $\bar{K}$, the agent has an incentive to report a higher than actual investment cost in order to obtain a higher (gross) compensation. This implies that the principal has to make untruthful reporting unattractive to the agent. The principal does so by delaying investment until the discount factor associated with the compensation corresponding to a false report $\hat{K}$ is sufficiently small. Consequently, the optimal choice of $\bar{x}_{\text{opt}}(K)$ ensures that the present value of the compensation based on untruthful reporting no longer exceeds the compensation based on truth-telling. Then, optimal (gross) compensation function $C(\hat{K})$ satisfies

$$
K = \arg\max_{\hat{K}} \left( C(\hat{K}) - K \left( \frac{x}{x_{\text{opt}}(\hat{K})} \right)^{\beta_1} \right).
$$

(5)

Compensation function $C(K)$ satisfying (5) and consistent with (4) increases with $K$. The informational rent is the highest for $K = \bar{K}$ and the gap between $C(K)$ and $K$ decreases with $K$ to reach zero for $K = \bar{K}$.

The investment inefficiency disappears only if the investment cost is equal to $\bar{K}$. In such
a case, the agent realizes the highest informational rent. For higher realizations of $K$, it is more beneficial for the principal to delay the investment (to make untruthful reporting less attractive due to a lower present value of the compensation) than to keep the informational rent at a constant (high) level. As a result, the informational rent of the agent decreases and the investment delay relative to the first-best investment policy increases with $K$. \textsuperscript{13}

Adverse selection is not the only problem that can emerge from the informational asymmetry between the principal and the agent. It is also possible that the principal observes the true investment cost but its level is influenced by an unobservable agent's effort. Grenadier and Wang (2004) analyze a situation in which there are two possible levels of the investment cost, $K$ and $\overline{K}$, and the probabilities of each of them occurring depend on the amount of effort.\textsuperscript{14} Assuming that exerting the high level of effort is socially desirable, the principal constructs a compensation scheme which is compatible with such a level of effort:

\begin{equation}
(C(K) - K) (p_h - p_l) \left( \frac{x_l}{x(K)} \right)^{\beta_l} = c(e_h) - c(e_l),
\end{equation}

where $p_h$ ($p_l$) is the probability that investment cost is $K$ when the level of effort is $e_h$ ($e_l$). Furthermore, $p_h$ ($e_h$) is greater than $p_l$ ($e_l$), and $c(\cdot)$ is an increasing function. Since the agent has to be provided with ex ante incentives only, the investment threshold is equal to the first-best threshold $\overline{x(K)}$. This implies that moral hazard does not affect the efficiency of the investment decision as it only affects the distribution of payoffs.

It is also possible that both adverse selection and moral hazard are present. Now, incentive compatibility is required not only ex post (truth-telling selection) but also ex ante (effort choice). Grenadier and Wang (2004) show that magnitude of adverse selection-driven underinvestment is lower in the presence of moral hazard. In other words, moral hazard reduces investment inefficiency resulting from the hidden information problem. This is due to the fact that the compensation which the agent receives under moral hazard is more closely linked to the
value of the investment project than in the absence if the hidden action component. Finally, if the cost of effort is sufficiently high, the compensation of the agent is so much aligned to the value of the firm that adverse selection does not occur anymore and the efficiency of investment policy is restored.

6 The exercise of collectively held options

This refers to a fairly general type of problem where two parties can generate a surplus by jointly exercising an option. The key issue is that the option can only be exercised if both parties agree on when to exercise and how to share the proceeds. This problem is analyzed in Lambrecht (2004) for the case of a merger between two firms (each run by an owner-manager). In his model the surplus (“synergies”) that firms receive from merging is generated by economies of scale, i.e., firms can produce by combining their production facilities produce more than the combined production when they each operate individually. The extra units of output that are created by merging can be sold at a stochastic product price. The surplus from merging is therefore not only stochastic, but it is also procyclical: it rises (falls) in periods of high (low) product market demand.

Upon merging each firm incurs a fixed merger cost. Since this is a sunk cost it introduces an element of irreversibility into the merger, and the decision to merge is therefore similar to the exercise of a call option. When merging, both companies therefore have to trade off the stochastic benefit of merging against the cost of merging. Since both firms have the right, but not the obligation to merge, each firm's payoff resembles an option and the decision to merge resembles the exercise of an option. The higher profits that firms forgo by not merging act as an incentive to exercise this option, while the (at least partially) irreversible nature of the merger acts as an incentive to delay. The optimal merger timing strikes a balance between the two.

Since the gains from mergers motivated by economies of scale are positively correlated to product market demand, mergers happen in rising product markets. This creates merger activity
at high output prices and merger inactivity at low output prices. Cyclical product markets will therefore generate a pattern of merger waves with mergers being procyclical.

The first-best (efficient) merger time is the one that would be chosen by a global optimizer that runs both firms and optimizes the global benefit from merging. This optimal output price level at which the merger takes place requires the size of the merger surplus (and hence the value of the new combined entity) at the time of the merger to be sufficiently large to cover the fixed costs of merging as well as the costs of giving up the cash flows that correspond to the existing two stand-alone firms.

The paper then examines whether mergers take place at the efficient time when both firm act in a non-cooperative way. Each party holds a call option on a fraction of the enlarged firm (i.e., the ownership share each party has in the new, combined firm) with as strike price the sum of its fixed merger cost and the standalone value of its existing firm. The merger can only take place if both parties agree on the timing of the merger (i.e., at what product price level should the merger happen) and the terms of the merger (i.e., what is the post-merger ownership share of each party in the new firm).

It follows that, unlike financial options, the exercise of ‘merger options’ is also influenced by strategic considerations since the payoff to each firm ultimately depends on the post-merger ownership share it obtains in the new firm. The restructuring mechanism (i.e., how the merger gains are divided up) can therefore also influence the timing of the restructuring. The paper considers two different mechanisms, namely friendly mergers and hostile takeovers, and shows that ceteris paribus they should occur at different stages in a merger wave. With friendly mergers it is in both parties interest first to maximize the total 'pie' to be divided, and subsequently to argue about how to divide the pie. It follows from Coase theorem that in the absence of frictions it is in the interest of both parties to adopt the globally efficient merger threshold. The second round negotiation problem then boils down to identifying the merger terms that induce both firms to exercise their merger option at the globally efficient merger threshold. Lambrecht (2004) shows that when firms are risk neutral there exists a unique Pareto
optimal sharing rule that induces both firms to execute the merger at the globally optimal time. He shows that each firm's post-merger ownership share is increasing in its size and its cost of merging.

The paper subsequently examines the case of hostile takeovers where negotiation about the surplus between the bidder and the target is not possible. In particular, the case is considered where the target credibly pre-commits to the terms it requires for relinquishing control, and the acquirer subsequently decides on the timing of the restructuring given those terms. The target's increased bargaining power enables it to charge the acquirer an additional bid premium. This bid premium acts as an added cost to the acquirer, raising the exercise price of the acquirer's takeover option, and causing hostile takeovers to take place later than mergers. The bid premium is increasing in the product market uncertainty and the bidder to target size ratio. Moreover, the higher the economies of scale that motivate the takeover, the stronger will be the size effect.

Morellec and Zhdanov (2005) extend the above model for the timing of takeovers by introducing multiple bidders and incomplete information. Competition between bidders may increase the bid premium and speed up the acquisition.

7 Conclusion

This paper considered a number of real options within a continuous-time corporate finance context. We analyzed whether these options are exercised efficiently, and what the underlying sources of inefficiencies are.

We showed that in some cases inefficiencies may be due to exogenously given constraints or market imperfections. In particular, when investors have to search for investment opportunities and face uncertainty as to the availability of investment opportunities in the future, then investment may take place inefficiently early. When making their investment decision investors trade off the benefit of waiting against the cost of not finding future investment
opportunities. Similarly, when investors may face future financing constraints, then they may prefer to invest inefficiently early instead of postponing and taking the risk of being unable to finance the project in future.

Another important source of inefficiency is conflicts of interest between investors. In some cases these conflicts may be extreme in that two investors may be competing for the same market, with the first mover being at an advantage. When investors have complete information they try to ‘epsilon’ preempt each other and consequently investment may happen inefficiently early. The inefficiency may be mitigated if both investors face incomplete information as to each other’s investment cost. In that case, investors make a tradeoff between the benefit of waiting and the cost of being preempted. The investors’ decision rule under preemption and incomplete information shares therefore similarities with the one adopted when there are search costs.

In many cases conflicts of interest may not be as extreme as in the case of two competing firms. For example, stockholders and bondholders of a firm very much have the same interests, especially when the firm is doing well. However, conflicts of interest may arise when the firm is in financial distress and the relationship between stockholders and bondholders may come to an end. For example, stockholders may be reluctant to contribute more capital because of the debt overhang problem, leading to underinvestment and inefficiently early closure. Bankruptcy costs may further exacerbate the agency problem since they are borne primarily by the bondholders, whereas the decision to pull the plug is often taken by the stockholders. Furthermore stockholders may have an incentive to take excessive risk in the run-down to bankruptcy (the asset substitution problem) since they have limited liability.

Similar conflicts of interest may arise between managers and outside equityholders. For example, private benefits of control may induce managers to overinvest and to close the firm inefficiently late. Also, the fact that equityholders' property rights are more protected than their cash-flow rights may create an incentive for managers to disinvest inefficiently late. Costs of collective action may often discourage stockholders to take punitive action against managers
that do not act in the best interest of stockholders. Asymmetry of information is another
important reason why managers can get away with taking suboptimal decisions. For example,
managers may not disclose the true value of the investment cost, in which case adverse selection
may take place. Alternatively, managers may not put in the optimal level of effort or they may
fail to make the most appropriate investment decision, creating a moral hazard problem.

Finally, we discussed the case of options that are held collectively, and where investors
have to agree on the timing of the option exercise, and on the way the exercise proceeds are
shared. We considered the case of two companies deciding on the timing and terms of a merger.
In the absence of market frictions firms should be able to achieve the efficient outcome.
However, negotiation costs, asymmetric information or bidding situations may lead to
inefficiencies.
Appendix

In this section we present the standard investment model as described in Dixit and Pindyck (1994), Ch. 6. The basic problem is to find the optimal timing of an irreversible investment, $K$, given that the cash flow from the project follows a geometric Brownian motion (GBM):

$$dx_i = \mu x_i dt + \sigma x_i dw_i.$$  \hspace{1cm} (7)

Parameter $\mu$ denotes the deterministic drift, $\sigma$ is the instantaneous standard deviation, and $dw$ is the increment of a Wiener process.

The deterministic riskless interest rate is $r$ and the drift rate $\mu$ satisfies $\mu < r$ so that finite valuations can be obtained. The firm is risk-neutral (or the spanning assumption holds) and it maximizes the value of the investment option, $F(x_i)$, by choosing the threshold value of $x_i$ at which the project is undertaken.

Since there are no intermediate payoffs to the holder of the investment option, the Bellman equation in the continuation region (i.e. before exercising the option) can be written as

$$rFdt = E[dF(x_i)].$$ \hspace{1cm} (8)

Equation (8) means that for a risk-neutral firm, the expected rate of change in the value of the investment opportunity over the time interval $dt$ equals the riskless rate. Applying Itô’s lemma to the RHS of (8), and dividing both sides of the equation by $dt$ results in the following ordinary differential equation (ODE):

$$rF = \mu x \frac{\partial F}{\partial x} + \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 F}{\partial x^2}.$$ \hspace{1cm} (9)

The general solution to (9) has the following form:

$$F(x) = A_1 x^\beta_1 + A_2 x^\beta_2,$$ \hspace{1cm} (10)

where $A_1$ and $A_2$ are constants, and
\begin{equation}
\beta_{1,2} = -\frac{\mu}{\sigma^2} + \frac{1}{2} \pm \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}}.
\end{equation}

Moreover, it holds that $\beta_1 > 1$ and $\beta_2 < 0$. In order to find the value of the investment option, $F(x_i)$, and the optimal investment threshold, $\bar{x}$, the following boundary conditions are applied to (10):

\begin{align*}
F\left(\bar{x}\right) &= \frac{\bar{x}}{r - \mu} - K, \\
F\left(\bar{x}\right) &= \frac{1}{r - \mu}, \\
F(0) &= 0.
\end{align*}

(12)

Conditions in the first and the second row of (12) are called the value-matching and the smooth-pasting conditions, respectively, and ensure continuity and differentiability of the value function at the investment threshold. Condition in the third row of ensures that the investment option is worthless at the absorbing barrier $x_i = 0$. Consequently, it implies that $A_2 = 0$.

Substitution of (10) into (12) and some algebraic manipulation yield the value of the optimal investment threshold:

\begin{equation}
\bar{x} = \frac{\beta_1}{\beta_1 - 1} K (r - \mu).
\end{equation}

(13)

Since $\beta_1 > 1$, the optimal investment threshold is strictly larger than 1 (cf. NPV rule). This reflects the value of waiting associated with the uncertainty of the project's value and the irreversibility of the investment decision. The value of the option to invest, $F(x)$, is given by

\begin{equation}
F(x_i) = \left(\frac{\bar{x}}{r - \mu} - K\right) \left(\frac{x_i}{\bar{x}}\right)^{\beta_1},
\end{equation}

(14)

where $\frac{\bar{x}}{r - \mu} - K$ is an NPV of the project at the moment of undertaking the investment. The second factor is a stochastic discount factor which equals the present value of $\$1$ received when the cash flow process hits the investment threshold $\bar{x}$. 

28
References


University Press.


1In the context of market exit, inefficiently early (late) abandonment corresponds to underinvestment (overinvestment).

2The standard real options investment threshold is derived in the appendix, where coefficient $\beta_i$ is also defined. The derivation is based on the seminal papers by Brennan and Schwartz (1985) and McDonald and Siegel (1986).

3If the first mover's advantage were not absolute in the sense that subsequent entry is still possible, then when both firms are identical, entry would occur at the point where both firms are indifferent between being the leader or the follower. This case is worked out in Smets (1993), Grenadier (1996), and Huisman and Kort (1999). The amount of option value that is destroyed would depend on how large the first mover's advantage is.

4Therefore, the option to invest in a costly search model can be interpreted as a Bermudan option with stochastic (and arriving according to a Poisson process) exercise dates.

5In a similar spirit, Hassett and Metcalf (1999) consider a model with an investment tax credit ceases to operate with a Poisson arrival rate $\lambda(x_i)$. As a result, investment occurs at the point at which the marginal benefit of waiting for a higher value of $x_i$ is exactly offset by the expected loss in the value of investment opportunity due to the tax credit withdrawal. This implies that investment occurs too early compared with the case with no uncertainty about taxes.

6The (dis)investment policy of a firm facing financing constraints can be enhanced by hedging. Mello and Parsons (2000) analyze the optimal hedging strategy preventing the firm from a premature liquidation, whereas Boyle and Guthrie (2004) develop a strategy that hedges liquidity necessary to optimally exercise the investment opportunity.

7The underlying rationale for this inequality is that the firm can borrow money against $A$ and $\alpha V_i$, which can then be used to finance the investment.

8The assumption about costless switching is waived in Dangl and Lehar (2004), whereas
Ericsson (2000) does not allow for more than one switch.  

The regulator's closure trigger is the upper bound of the random audit region, the lower bound being the equityholders' endogenous exit threshold.  

An alternative way to model the principal-agent problem would be to assume that manager controls the drift of the process underlying the project value. In a classic contribution Holmstrom and Milgrom (1987) prove the optimality of a linear contract when the investor's objective is to maximize the terminal value of the project. More recently, Kenc, Mella-Barral and Perraudin (1996) apply an infinite-horizon framework and look at how the distribution of the bargaining power between the manager who controls the drift and shareholders affects the value of the firm and the liquidation policy.  

It has to be assumed that ratio \( \frac{\Phi(K)}{\phi(K)} \) increases with \( K \), which is a standard assumption in the mechanism design literature.  

Such a higher threshold is consistent with CEO survey evidence that firms may require higher hurdle rates of returns that those predicted by finance theory (cf. Poterba and Summers (1995)).  

Mæland (2001) extends the basic model of investment with adverse selection by allowing for \( i \) a stochastic development of the investment cost \( K \), and \( ii \) the presence of multiple agents who offer their services in an auction. Allowing for the stochastic evolution of the investment cost does not change the form of the solution apart from the fact that ratio \( \frac{\Phi(K)}{\phi(K)} \) becomes stochastic. Introducing multiple agents, whose offers are chosen in an auction, leads to a lower average cost of investment.  

In fact, the variable component in Grenadier and Wang (2004) is a premium on the top of the value of project \( V' \), which is equivalent to the discount in investment costs. We choose the latter interpretation to make the framework of Grenadier and Wang (2004) comparable with Mæland (1999).