# Real Options in an Asymmetric Duopoly: Who Benefits from Your Competitive Disadvantage?*

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**Abstract**

This paper analyzes the impact of investment cost asymmetry on the optimal real option exercise strategies and the value of firms in duopoly. Both firms have an opportunity to invest in a project enhancing (ceteris paribus) the profit flow. We show that three types of equilibrium strategies exist. Furthermore, we express the critical levels of cost asymmetry delineating the equilibrium regions as functions of basic economic variables. The presence of strategic interactions among the firms leads to counterintuitive results. First, for a certain range of the asymmetry level, a marginal increase in the investment cost of the firm with the cost disadvantage can enhance this firm's own value. Moreover, such a cost increase can reduce the value of the competitor. Finally, we discuss the welfare implications of the optimal exercise strategies and show that the presence of identical firms can result in a socially less desirable outcome than if one of the competitors has a significant cost (dis)advantage.

# 1 Introduction

The aim of this paper is to study the effects of investment costs asymmetry on the optimal real option exercise strategies and the value of duopolistic firms. The need for a separate treatment of real, as opposed to financial, options results from the non-exclusivity of the former. In particular, the optimal exercise decision of a firm competing in an oligopolistic market depends not only on the value of the underlying economic variable but also on the actions undertaken by its competitor(s) (cf. for instance, Myers, 1987, Smets, 1991, and Zingales, 2000). For example, the investment opportunity to set up the production of a new patented and FDA-approved drug in place of the existing product may be represented as a real option to exchange a fixed amount of money for an incremental stream of uncertain cash flows. The net cost of switching the production line from the old drug to the newly developed one can be viewed as the strike price of the option, whereas the increase in the expected present value of the stochastic profit flow corresponds to the underlying asset. The value of the investment opportunity, as well as the optimal exercise strategy associated with it, highly depends on the actions taken by the competing drug vendor. Consequently, neither the value of the firm nor the optimal exercise strategy resembles any longer the situation where the firm has an exclusive option to invest.

The last decade's research results in a number of contributions dealing with the non-exclusivity of real options. The basic continuous-time model of strategic real option exercise under product market competition is presented by Smets (1991). He considers a duopolistic firm's decision to (costly) switch the production from a developed to an emerging economy where production costs are lower. Applications and extensions of the strategic real option exercise model include Boyer, Lasserre, Mariotti and Moreaux (2002), Décamps and Mariotti (2000), Grenadier (1996, 1999), Huisman and Kort (1999), Lambrecht and Perraudin

(2003), Mason and Weeds (2003), Perotti and Rossetto (2000), Shackleton, Tsekrekos and Wojakowski (2004), and Williams (1993).[1] Earlier contributions by Fudenberg and Tirole (1985), and Reinganum (1981) provide the game-theoretical foundations within a deterministic framework. Tirole (1988), Ch. 8 provides an overview of the related entry deterrence models, whereas Shaked and Sutton (1990) consider a product market structure as a function of, among others, the costs of entry.

In this paper we analyze the situation where two firms have an opportunity to invest in a profit enhancing investment project and face different (effective) investment costs. This framework, which relaxes the restrictive assumption that the duopolistic rivals are identical, is motivated by the existence of many sources of potential cost asymmetry. The choice of the investment cost asymmetry as an approach to incorporate the difference between firms can be motivated by a number of factors.[2]

First, investment cost asymmetry is present when the firms have different access to the capital markets. In such a case, the cost of capital of a liquidity-constrained firm is higher than of its counterpart having access to a credit line or with substantial cash reserves (Lensink, Bo and Sterken, 2001). Consequently, the investment cost of the firm facing capital market imperfections is higher as well.

Moreover, cost asymmetry occurs when the firms exhibit a different degree of organizational flexibility at implementing a new production technology. This flexibility, known as absorptive capacity (cf. Cohen and Levintal, 1994), measures the firm's ability to adopt external technologies, to assimilate to a changing economic environment, and to commercialize newly invented products. A higher absorptive capacity is therefore equivalent to a lower cost associated with an investment project.

Differing real options embedded in the existing assets of the firms due to past decisions are another source of possible investment cost asymmetry. After the arrival of a new invention it may appear that one of the existing technologies is more easily extendable than the other. For instance, Kaplan (1986) reports that in the 1970s some manufacturing firms invested in electronically controlled production facilities. This investment did not bring significant improvements to the firms' profits. However, after the arrival of microprocessor-based technology in the 1980s, the firms that invested in electronically controlled facilities were able to adopt the new technology more quickly and at a lower cost.

Finally, the difference in investment costs is often a consequence of purely exogenous factors, resulting, among others, from the intervention of the authority. For instance, the effective investment cost of the firms is reduced after obtaining governmental credit guarantees, which result in a lower cost of capital (see the evidence by Kleimeier and Megginson, 2000).

Another objective of our paper is to generalize a number of existing strategic real options contributions into a single, unified framework. We consider the optimal real option exercise strategy of duopolistic firms and allow them to already compete in a product market. Both firms have an investment opportunity enhancing (ceteris paribus) the profit flow. If one firm invests, the other firm's payoff is reduced. This is, for example, the case when the investment gives the firm the possibility to produce more efficiently and thus more cheaply, which leads to a higher market share. The firms differ ex ante only with respect to the required sunk cost associated with the investment. The limiting cases of our framework are Smets (1991) and Grenadier (1996), who restrict the analysis to a game between symmetric firms. After setting the initial profits to zero, our model reduces to Huisman (2001), Ch. 8, and Joaquin and Butler (2000), who consider a new market entry of asymmetric firms. By letting the initial profits be higher than the profits after mutual investment we arrive at

Perotti and Rossetto (2000).

Our framework results in the presence of three different equilibrium strategies. First, when the asymmetry among firms is relatively small and so is the first-mover advantage, the firms invest at the same time. Second, when the first-mover advantage is sufficiently large, the low-cost firm preempts the high-cost firm. Third, in the situation where both the first-mover advantage and asymmetry between firms are significant, the firms exercise their investment options sequentially. In such a situation, one firm's investment timing does not directly affect the investment decision of its competitor.[3] We obtain that higher uncertainty delays investment not only by increasing trigger values of the economic variable at which investment is undertaken but also by favoring those strategies that are associated with a relatively late investment (preemption vs. tacit collusion).

We also determine the firms' values and present welfare implications of the strategic real option exercise. We find that, when an increase in the investment expenditure of the high-cost firm results in a switch from the joint investment to the preemptive equilibrium, the value of *both* firms decrease. Furthermore, an increase in the high-cost firm's investment expenditure in the preemptive equilibrium can result in the increase of the firm's own value. This is due to the fact that the low-cost firm facing now a weaker competitor can delay the investment without bearing the risk of being preempted. This investment delay raises the value of the high-cost firm as well.

Using an example of a duopoly where after investment the firms offer a product of a higher quality, we derive the relationship between the type of equilibrium and the level of social welfare. Our analysis indicates that an equal access of competitors to a new technology (or a new market) is in some economically plausible scenarios not socially optimal. This holds if the required investment cost is small compared to the incremental consumer surplus that can be attributed to the new investment. In such cases, the policies aimed at stimulating investment of the potential first mover are superior from the welfare maximization viewpoint to the policies with investment stimuli distributed evenly across firms. This is due to the fact that the prospect of a temporary monopolistic position of the (low-cost) firm leads to its earlier investment, which is beneficial from the welfare standpoint. This finding, which is in line with Schumpeterian view, provides an alternative explanation of the positive relationship between the presence of monopolistic quasi-rents and the pace of innovation (represented in our model by investment). Furthermore, we show that higher uncertainty reduces the probability of such a scenario.

The paper is organized as follows. In Section 2 we present the model. Section 3 contains the derivation of value functions and optimal investment thresholds. The discussion of the resulting equilibrium strategies is presented in Section 4. In Section 5 we analyze the impact of strategic interactions on the value of the firms, whereas Section 6 discusses the relationship between the firms' investment strategies and social welfare. Section 7 concludes.

## 2 Model

Essentially, the basic framework of Dixit and Pindyck (1996), Ch. 6, is adapted here, with the difference that we consider two firms rather than one. The two risk-neutral firms compete in the product market and have a single investment opportunity to enhance their profit flow.[4] The uncertainty in each of the firms' profits is introduced via a geometric Brownian motion:

$$dx\left(t\right)=\alpha x\left(t\right)dt+\sigma x\left(t\right)dw, \tag{1}$$

where $\alpha$ and $\sigma$ are constants corresponding to the instantaneous drift and to the instantaneous standard deviation, respectively, $dt$ is the time increment and $dw$ is the Wiener increment, which is normally distributed with mean zero and variance $dt$. Consequently, parameters $\alpha$ and $\sigma$ can be interpreted as the industry growth rate and the industry volatility, respectively. The riskless interest rate equals $r$. In order to obtain finite valuations we assume that $\alpha < r$. The instantaneous profit of Firm $i$, $i \in \{1, 2\}$, can be expressed as

$$\pi_{N_i N_j}(x) = x D_{N_i N_j}, \tag{2}$$

where, for $k \in \{i, j\}$ :

$$N_k = \begin{cases} 0 & \text{if firm } k \text{ has not invested,} \\ 1 & \text{if firm } k \text{ has invested.} \end{cases}$$

$D_{N_i N_j}$ stands for the deterministic contribution to the profit function, and it holds that

$$\begin{array}{ccc} D_{10} & > & D_{00} \\ \vee & & \vee \\ D_{11} & > & D_{01}. \end{array} \tag{3}$$

$D_{10} > D_{00}$ implies that the profit of the firm that invests as first exceeds *ceteris paribus* the initial (symmetric) profit. This inequality also implies that the profit flow increment resulting from the new investment exceeds the cannibalization effect of investment on the profits from the existing assets. Furthermore, this investment leads to a deterioration of the profit of the firm that did not undertake the project yet, i.e. $D_{00} > D_{01}$. Finally, the 'me-too' investment made by the lagging firm enhances its profit, so $D_{11} > D_{01}$, but, at the same time, it reduces the profit of the first mover, so that $D_{11} < D_{10}$. Such a general formulation embraces, for instance, Cournot or Stackelberg quantity competition.[5]

The investment opportunity is assumed to last forever and the structure of the associated payoff can only change as a result of the competitor's action. Furthermore, we denote the investment cost of Firm $i$ by $I_i$. Without loss of generality $I_1$ is normalized to $I$, and $I_2$

is defined as

$$I_2 \equiv \kappa I, \tag{4}$$

where $\kappa \in (1, \infty)$. Consequently, Firm 1 is the low-cost firm and the high-cost firm is labelled as Firm 2.

Finally, we assume that the initial realization of the process underlying both firms' profits, $x(0)$, is low enough, so that an immediate investment is not optimal.[6]

# 3 Value Functions and Investment Thresholds

There are three possibilities concerning the relative timing of the firms' investment. First, Firm $i$ can invest first, which means that it becomes the leader. Alternatively, Firm $j$ can invest sooner which results in Firm $i$ becoming the follower. Finally, the firms can invest simultaneously.

In this section we establish the payoffs associated with the three situations described above. As in the standard approach used to solve dynamic games, we analyze the problem backwards in time. First, we derive the optimal strategy of the follower, who takes the strategy of the leader as given. Subsequently, we analyze the decision of the leader. We are interested in the value functions of the leader and of the follower under the assumption that the leader invests immediately. This formulation allows us to compare the value of investing immediately (as the leader) with the value of waiting (and investing later as the follower). Such a comparison will make it possible to determine the optimal investment strategies (cf. Fudenberg and Tirole, 1985, which is the seminal reference for this concept).[7] After discussing the value functions of the follower and of the leader, we analyze the case of joint investment.

As we show later, introducing asymmetry between firms uniquely determines their roles. Firm 1 (the low-cost firm) always takes the role of the leader unless the firms invest simultaneously. In combination with the assumption that $x(0)$ is sufficiently small, this implies that the resulting firms' strategies have a simple form of threshold values of process (1) at which the investment is made.[8]

### 3.1 Follower

Consider the investment decision of the follower (say, Firm $i$) in a situation where the leader (in this case, Firm $j$) has already invested. Since after investing the leader takes no further action, the problem of the follower closely resembles a single-firm investment problem known from the real option theory. From this theory (cf. Dixit and Pindyck, 1996), we know that Firm $i$ will undertake the investment when profits are sufficiently large, i.e. when $x$ exceeds a certain threshold level, which we denote by $x_i^F$. This optimal investment threshold is calculated by applying the standard dynamic programming methodology (see Appendix A) and equals

$$x_i^F = \frac{\beta}{\beta - 1} \frac{I_i (r - \alpha)}{D_{11} - D_{01}}, \tag{5}$$

where $\beta$ is given by

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1. \tag{6}$$

By rewriting (5) as

$$\frac{x_i^F (D_{11} - D_{01})}{r - \alpha} = \gamma(\beta) I_i, \tag{7}$$

where $\gamma(\beta) \equiv \beta / (\beta - 1)$, it becomes straightforward that the optimal investment threshold is a modified net present value (NPV) formula with mark-up $\gamma(\beta)$, which is larger than 1. This mark-up reflects the impact of irreversibility and uncertainty and is increasing in uncertainty (since $\partial\beta/\partial\sigma < 0$). The latter result is in line with real options theory according to which the value of waiting to invest is higher in a more uncertain economic environment.

From the analysis leading to the optimal investment threshold $x_i^F$, it also follows that the value of Firm $i$ as the follower equals

$$V_i^F(x) = \begin{cases} \frac{xD_{01}}{r-\alpha} + \left(\frac{x_i^F(D_{11}-D_{01})}{r-\alpha} - I_i\right)\left(\frac{x}{x_i^F}\right)^{\beta} & \text{if} \quad x \leq x_i^F, \\ \frac{xD_{11}}{r-\alpha} - I_i & \text{if} \quad x > x_i^F, \end{cases} \tag{8}$$

The first row of (8) is the present value of profits when the follower does not invest immediately. Its first term is the payoff in case the follower refrains from ever investing, whereas the second term equals the value of the option to invest. The latter equals the NPV of the follower's investment discounted back from the (random) time of reaching the follower's threshold $x_i^F$. Consequently, $\left(x/x_i^F\right)^{\beta}$ is interpreted as a stochastic discount factor, which is equal to the present value of \$1 received when process (1) hits the investment threshold $x_i^F$ when starting from level $x < x_i^F$. This discount factor is negatively related to interest rate $r$ and positively depends on drift rate $\alpha$ and uncertainty $\sigma$. The second row of (8) corresponds to the NPV of the profit flow when immediate investment is optimal.

### 3.2 Leader

Having established the optimal investment threshold of the follower, we are in position to determine the payoff of Firm $i$ when it invests immediately as the leader. The value function of Firm $i$, evaluated at the moment of investing equals

$$V_i^L(x) = \begin{cases} \frac{xD_{10}}{r-\alpha} - I_i - \frac{x_j^F(D_{10}-D_{11})}{r-\alpha}\left(\frac{x}{x_j^F}\right)^{\beta} & \text{if} \quad x \leq x_j^F, \\ \frac{xD_{11}}{r-\alpha} - I_i & \text{if} \quad x > x_j^F. \end{cases} \tag{9}$$

The first row of (9) is the NPV of the leader's profits if the follower (Firm $j$ in this case) does not invest immediately. This NPV, via its last component, takes into account the present value of future profits lost due to the follower's investment at $x_j^F$. The second row corresponds to the net present value of profits in a situation where it is optimal for the follower to invest immediately.

### 3.3 Simultaneous Investment

It is possible that the firms, despite the asymmetry in their investment costs, decide to invest simultaneously. The value function of Firm $i$ when investing simultaneously with Firm $j$ as soon as $x$ exceeds some threshold $x^S$ is

$$V_i^S\left(x;x^S\right)=\begin{cases}\frac{xD_{00}}{r-\alpha}+\left(\frac{x^S(D_{11}-D_{00})}{r-\alpha}-I_i\right)\left(\frac{x}{x^S}\right)^\beta & \text{if} \quad x\leq x^S,\\ \frac{xD_{11}}{r-\alpha}-I_i & \text{if} \quad x>x^S.\end{cases} \tag{10}$$

The first row of (10) consists of the present value of Firm $i$ based on its existing cash flow plus the option to enhance the instantaneous profit (from $xD_{00}$ to $xD_{11}$) at threshold $x^S$. The second row equals the value of Firm $i$ when the simultaneous investment is made immediately.

For each firm, such value of $x^S$ exists that maximizes the value of its investment opportunity. Such an optimal threshold of Firm $i$ when the firms invest simultaneously is given by

$$x_i^S=\frac{\beta}{\beta-1}\frac{I_i\left(r-\alpha\right)}{D_{11}-D_{00}}. \tag{11}$$

Again, as in the case of the follower's threshold (cf. (5)), the relationship between $\sigma$ and $x_i^S$ is positive. The simultaneous investment threshold exists as long as $D_{11}$ is larger than $D_{00}$. Otherwise, it is optimal for the firms to abstain from investing.[9] From (11) it can be seen that $x_i^S$ differs from $x_j^S$ due to differing investment costs. As it is shown in the next section, this divergence does not preclude the simultaneous investment strategy.

The value of Firm $i$ when both firms invest at $x_i^S$ is denoted by $V_i^S\left(x\right)$.

## 4 Equilibria

There are three types of equilibria that can occur in the choice of firms' strategies, namely the preemptive, sequential and simultaneous equilibrium. In this section we characterize each type of equilibrium and present the conditions under which each of them occurs.

### 4.1 Preemptive Equilibrium

The first type of equilibrium that we consider is the preemptive equilibrium. It occurs when both firms have an incentive to become the leader, i.e. when the cost disadvantage of Firm 2 is relatively small. Therefore, Firm 1 has to take into account the fact that Firm 2 will aim at preempting it as soon as a certain threshold is reached. This threshold, denoted by $x_{21}^P$, is the lowest realization of the process $x$ for which Firm 2 is indifferent between being the leader and the follower. Formally, $x_{21}^P$ is the smallest solution to $\xi_2(x) = 0$, where $\xi_i(x)$ is defined as

$$\xi_i(x) \equiv V_i^L(x) - V_i^F(x). \tag{12}$$

$V_i^F(x)$ and $V_i^L(x)$ are given by (8) and (9), respectively. Firm 1 invests at $\min\left(x_{21}^P, x_1^L\right)$, where

$$x_1^L = \frac{\beta}{\beta-1}\frac{I(r-\alpha)}{D_{10}-D_{00}} \tag{13}$$

is Firm 1's optimal leader threshold when the roles of the firms are pre-determined (i.e. preemption by Firm 2 is excluded).[10]

Figures 1 and 2 illustrate the firms' payoffs associated with being the leader, the follower, and with both investing at Firm 1's optimal simultaneous investment threshold when the resulting equilibrium is of the preemption type.

[Please insert Figure 1 about here]

[Please insert Figure 2 about here]

The reason why Firm 1 invests at $\min\left(x_{21}^P, x_1^L\right)$ is the following. If $x_1^L < x_{21}^P$, the follower value of Firm 2 exceeds its leader value for all $x < x_1^L$. In such a case, unchallenged Firm 1 maximizes the value of its investment opportunity by investing at its optimal threshold $x_1^L$. In the opposite situation, i.e. when $x_1^L \geq x_{21}^P$ (which is the case depicted in Figures 1 and 2), Firm 1 would still optimally invest as the leader at its optimal threshold $x_1^L$. However, Firm 2 anticipates this and, since its leader value at $x_1^L$ is larger than its follower value, it has an incentive to "epsilon-preempt" Firm 1 and invest at $x_1^L - \varepsilon$. Then, the reaction of Firm 1 would be to invest at $x_1^L - 2\varepsilon$ and so on. The resulting preemption game leads to an outcome in which Firm 1 invests at Firm 2's preemption point $x_{21}^P$. This is due to the fact that for $x < x_{21}^P$ Firm 2 no longer wants to be the leader (as its leader value is lower than the value as the follower). Moreover, Firm 2 does not invest immediately after Firm 1 does since in such a case both firms would realize the low payoff $V_i^S\left(x_{21}^P; x_{21}^P\right)$. Instead, Firm 2 invests optimally as the follower at $x_2^F$.

**Proposition 1** *In the preemptive equilibrium, Firm 1 extracts a relative surplus from becoming the leader vs. being the follower, i.e.*

$$\xi_1\left(\min\left(x_{21}^P, x_1^L\right)\right) = V_1^L\left(\min\left(x_{21}^P, x_1^L\right)\right) - V_1^F\left(\min\left(x_{21}^P, x_1^L\right)\right) > 0. \tag{14}$$

**Proof.** See Appendix B. ■

The presence of cost asymmetry implies that there is no rent equalization between the leader and the follower (cf. Fudenberg and Tirole, 1985). Consequently, Firm 1 is not indifferent between these two roles in the preemptive equilibrium. Moreover, it holds that even for the slightest degree of cost asymmetry, it is Firm 1 that always takes the leader's role.

This simple and more intuitive identification of the equilibrium strategies with the low-cost firm investing first is also present in Huisman (2001), Joaquin and Butler (2000), and Perotti and Rossetto (2000).[11]

Finally, despite the presence of strategic interactions among the firms, it holds that higher uncertainty raises the leader's threshold.[12]

### 4.2 Sequential Equilibrium

The sequential equilibrium occurs when Firm 2 has no incentive to become the leader, i.e. when $\xi_2(x) < 0$ for all $x < x_2^F$. In this case, Firm 1 simply maximizes the value of its investment opportunity, which always leads to investment at the optimal threshold $x_1^L$. In other words, Firm 1 acts as if it had exclusive rights to invest in a profit-enhancing project (but, of course, Firm 2's investment still affects Firm 1's payoff). This result is due to the fact that the future competitive investment affects the value of the leader's investment opportunity to the same extent as it influences the value of its installed project.

The described situation closely resembles Leahy (1993), in which a firm which faces subsequent investments of its competitors pursues the same investment policy as a monopolist that is not threatened by such future events.[13] In fact, this argument implies the irrelevance of future investment opportunities of the firm and its competitor for the timing of the present investment. This is why a number of outcomes of the strategic model with multiple investment opportunities (see Boyer et al., 2002) can be obtained in a single investment opportunity framework as long as the latter allows for a sufficiently wide range of initial conditions of the game. (Of course, the irrelevance result ceases to hold if the market structure allows for more complex interactions, such as two consecutive preemptive investments by a single firm.)

Figure 3 illustrates Firm 2's payoffs associated with the sequential investment equilibrium.

[Please insert Figure 3 about here]

By comparing $V_2^L(x)$ and $V_2^F(x)$ in Figure 3, it can be concluded that Firm 2 is never better off by becoming the leader compared to being the follower. Therefore Firm 1 does not need to take into account the possibility of being preempted by Firm 2. As a result, Firm 1 is able to invest at threshold $x_1^L$, which maximizes the value of the leader's investment opportunity. As in the previous case, Firm 2 invests at its follower threshold $x_2^F$.

### 4.3 Simultaneous Equilibrium

Another type of equilibrium is the simultaneous equilibrium. In this case the firms invest at the same point in time. Although such joint investment is a non-cooperative outcome, it is often referred to as tacit collusion due to the fact that it implies *"some implicit coordination to increase rents over their preemption level"* (Boyer et al., 2002, p. 19). This type of equilibrium is only possible if firms already compete in the market and they face the risk of cannibalizing their profits from existing assets. Otherwise, the tacit collusion would not be sustainable since the threat of losing existing profits would not exist (as in Huisman, 2001, and Joaquin and Butler, 2000). Figure 4 depicts Firm 1's payoffs associated with the simultaneous equilibrium.

[Please insert Figure 4 about here]

Since firms face different investment costs, the simultaneous equilibrium does not maximize their joint value. This result contradicts the corresponding result for symmetric firms (cf. Huisman and Kort, 1999) and is due to the fact that the optimal joint investment thresholds (11) differ for both firms. (In fact, it can be shown that due to the proportionality of the thresholds to the investment cost, the joint value-maximizing threshold is the arithmetic average of the firms' individual thresholds.) Such deviation from the joint value-maximization in the tacit collusion equilibrium is also present in Boyer et al. (2002) in a situation where firms are of different sizes.

As the result of investment cost asymmetry, one of the firms has to adopt a strategy that does not optimize its payoff. Since the optimal threshold of Firm 1 is lower than that of Firm 2, the only candidate for a simultaneous investment threshold is $x_1^S$. For simultaneous investment to occur, the payoff of Firm 1 associated with being the leader has to be lower than the payoff resulting from simultaneous investment at $x_1^S$, i.e. $V_1^S(x) \geq V_1^L(x)$ for all $x$ (see Figure 4). Otherwise, Firm 1 will invest either at $x_1^L$ or at $x_{21}^P$, depending on the level of cost asymmetry. Moreover, Firm 2's follower threshold must be lower than $x_1^S$. In other words, Firm 2 has to find it more profitable to respond to Firm 1's investment at $x_1^S$ immediately than to wait. Otherwise, Firm 2 would invest as the follower at $x_2^F$. It turns out that whenever it is optimal for Firm 1 to invest simultaneously, Firm 2 prefers simultaneous investment at $x_1^S$ to being the follower (see the proof of Proposition 2 in Appendix B).

### 4.4 Conditions for Equilibria

The occurrence of a particular type of equilibrium is determined by the relationship between the relative payoffs, which in turn depends on the level of cost asymmetry, the first-mover advantage and market parameters such as volatility, the growth rate and the interest rate.

The following proposition defines the cut-off level of investment cost asymmetry that delineates the regions of preemptive and sequential equilibria.

**Proposition 2** *There exists a unique value of $\kappa > 1$, denoted by $\kappa^*$, which is equal to*

$$\kappa^* = \left( \frac{w^\beta - 1}{\beta (w-1)} \right)^{\frac{1}{\beta - 1}}, \tag{15}$$

*where*

$$w = \frac{D_{10} - D_{01}}{D_{11} - D_{01}}, \tag{16}$$

*which separates the regions of the preemptive and the sequential equilibrium, conditionally on one of them occurring. For $\kappa < \kappa^*$ Firm 1 needs to take into account possible preemption by Firm 2, whereas $\kappa \geq \kappa^*$ implies that firms always invest sequentially at their optimal thresholds.*

**Proof.** See Appendix B. ■

Proposition 2 states that there is a level of the Firm 2's cost disadvantage above which Firm 1 can act as a monopolist in exercising its investment option.[14] The cut-off level is a function of only two parameters: $w$ and $\beta$. Parameter $w$ is a measure of the first-mover advantage, which can result from improved product characteristics or greater cost-efficiency. Parameter $\beta$ (cf. equation (6)) captures the impact of the macroeconomic variables, such as interest rate, industry growth rate and industry volatility.

**Proposition 3** *There exists a unique value of $\kappa \geq 1$, denoted by $\kappa^{**}$, which is equal to*

$$\kappa^{**} = \max \left( v \left( \frac{\beta (u-1)}{u^\beta - 1} \right)^{\frac{1}{\beta - 1}}, 1 \right), \tag{17}$$

*where*

$$u = \frac{D_{10} - D_{00}}{D_{11} - D_{00}} \qquad \text{and} \qquad v = \frac{D_{11} - D_{01}}{D_{11} - D_{00}}, \tag{18}$$

*which determines the regions of the simultaneous and the sequential/preemptive investment equilibria. For $\kappa < \kappa^{**}$ the resulting equilibrium is of the joint investment type, whereas for $\kappa \geq \kappa^{**}$ the sequential/preemptive investment equilibrium occurs.*

**Proof.** See Appendix B. ■

According to Proposition 3, for a relatively high degree of asymmetry between firms simultaneous investment is not optimal so either a sequential or preemptive equilibrium occurs. Moreover, a set of parameter values exists for which simultaneous investment is not optimal even when the firms are symmetric. In this case $\kappa^{**}$ is equal to 1.

The cut-off level is a function of three parameters: $u$, $v$ and $\beta$. Parameters $u$ and $v$ characterize the product market structure: the former is a measure of the first-mover advantage and the latter represents the benefit of the second mover resulting from its 'me-too' investment. It is worthwhile emphasizing that the occurrence of either the preemptive/sequential or the simultaneous equilibrium in Boyer et al. (2002) is primarily determined by the maturity of the industry, whereas in our model it reflects the magnitude of the cost asymmetry.

Now, we are in position to illustrate the resulting equilibria in a two-dimensional graph. In Figure 5 the regions of investment strategies are depicted as a function of the first-mover advantage (defined here as the ratio of $D_{10}$ to $D_{11}$), and the investment cost asymmetry, $\kappa$. When the investment cost asymmetry is relatively small and there is no significant first-mover advantage, the firms invest jointly (the triangular area in the south-west). In the situation where the first-mover advantage becomes significant, Firm 1 prefers being the leader to investing simultaneously. This results in the preemptive equilibrium (the area in the south-east). Finally, if there is a large asymmetry between firms (i.e. when $\kappa > \max(\kappa^*, \kappa^{**})$), the firms invest sequentially and Firm 1 can act as a sole holder of the investment opportunity.

[Please insert Figure 5 about here]

By changing the values of parameter $\beta$ in (15) and (17) it can be shown that the critical value $\kappa^*$ ($\kappa^{**}$) decreases (increases) in uncertainty, which leads to reducing the area of the preemption region (see Figure 4). Therefore, uncertainty delays investment not only by raising investment thresholds in a given equilibrium type but also by possibly making the firms switch across equilibria. In the latter case, higher uncertainty can result in a switch from preemptive to the sequential equilibrium (via decreasing $\kappa^*$) and from preemptive/sequential to the simultaneous one (via increasing $\kappa^{**}$). Hence, whenever the switch occurs, it is a switch from an equilibrium where firms invest relatively soon to an equilibrium where firms invest late. This result is consistent with one of the main results of the standard real option theory: uncertainty creates a value of waiting with investment.

## 5 Cost Asymmetry and Value of the Firm

The magnitude of investment cost asymmetry has a significant impact on the value of each firm and, in particular, on the present value of their investment opportunities. We show that, due to the presence of strategic interactions, the relationship between the magnitude of the investment cost asymmetry and the value of the firm is, in general, discontinuous and non-monotonic.

In the absence of strategic interactions, the value-asymmetry relationship is relatively straightforward. An increase in the investment cost of Firm 2 affects its value via *i*) a higher present value of the investment expenditure that has to be incurred and *ii*) a delay in the investment which results in postponing the moment of the profit flow increase. Consequently, the value of Firm 2 decreases monotonically in $\kappa$. The value of Firm 1 remains unaffected by a change in $\kappa$ if the firms do not interact with each other.

Introducing imperfect competition changes the way the asymmetry affects the values of both firms. The value of Firm 2 is affected not only by an increase in its investment cost but also by the change of Firm 1's investment timing. This is due to the fact that Firm 1 invests later (i.e. at higher $x_{21}^P$) when asymmetry parameter $\kappa$ increases. In turn, the value of Firm 1 is positively affected by the delayed investment of Firm 2 and, in the preemptive equilibrium, by a weaker competitive pressure (in the sense of a less aggressive preemptive behavior of Firm 2). Firm 2 being a weaker competitor allows Firm 1 for a more efficient timing of its own investment. Finally, changing cost asymmetry can result in a switch across the equilibrium types, which results in the firm values becoming discontinuous and non-monotonic functions of $\kappa$.

We illustrate the impact of strategic interactions for the case where parameter values are chosen in such a way that for changing values of the cost asymmetry parameter all three types of equilibria are possible (cf. Figure 5). The firms' values resulting from their optimal strategies are depicted in Figure 6.[15]

[Please insert Figure 6 about here]

The lowest degree of asymmetry between the firms corresponds to the simultaneous investment equilibrium. In this equilibrium the outcome closely resembles the case where strategic interactions are absent. In other words, a marginal increase in $\kappa$ does not affect the value of Firm 1 and has a negative impact on the value of Firm 2 only via its higher investment cost.

As $\kappa$ increases, sequential investment becomes more attractive for Firm 1 because of the increasing Firm 2's follower threshold. A later investment of Firm 2 results in an increased Firm 1's payoff from sequential investment. Consequently, for $\kappa$ exceeding $\kappa^{**}$, Firm 1 would optimally invest at its leader threshold $x_1^L$. However, Firm 2 anticipates this and, as explained in Section 4.1, the preemption game results. Consequently, Firm 1 invests at Firm 2's preemption point $x_{21}^P < x_1^L$. The implication is that a marginal increase in the investment cost of Firm 2 at $\kappa^{**}$, which changes the equilibrium type from simultaneous to preemptive, results in both firms' values moving downward.

Once the firms are in the preemption region, the values of both of them increase with $\kappa$. Obviously, the value Firm 1 increases because its investment threshold moves closer to $x_1^L$. Moreover, it benefits from the delayed investment of Firm 2. A more surprising result is a positive relationship between Firm 2's value and its investment cost. It is caused by the fact that increasing $\kappa$ makes Firm 2 a weaker competitor. This means that the threat of Firm 2 making the preemptive investment decreases with the investment cost asymmetry. This implies that Firm 1 invests later, which positively affects the profit flow of Firm 2, that can now enjoy its higher level, $xD_{00}$, for a longer period. In such a case, the non-strategic (higher $I_2$) and the strategic (higher $x_{21}^P$) effects operate in the opposite directions and the latter effect dominates.[16]

When the asymmetry between the firms reaches the critical level $\kappa^*$, above which the value of Firm 2 as the follower always exceeds its leader value, the sequential equilibrium occurs. Upon the switch to the sequential equilibrium, the values of both firms move upward. In both cases this is caused by the discontinuous change, from $x_{21}^P$ to $x_1^L$, of Firm 1's investment threshold. By investing at $x_1^L$ Firm 1 maximizes its own value, and lets Firm 2 enjoy a higher profit flow for a longer period.

In the sequential equilibrium region the changes in the firms' values result entirely from the cost asymmetry and its impact on Firm 2's investment timing. Firm 1 benefits from the delayed investment of Firm 2 and the value of the latter decreases for the same reason as in the non-strategic case.

# 6 Welfare Analysis

In order to assess the desirability of policies influencing firms' access to new market segments and technologies, we investigate how investment cost asymmetry affects social welfare. The firms' investment costs can be influenced by the regulator, for instance, via fiscal measures, debt guarantees, and diffusion policies aimed at regulating knowledge spillovers.[17]

Having already determined the firms' values, we begin this section's analysis with deriving the consumer surplus. Once we are able to calculate social welfare, we analyze how it is affected by the choice of the firms' investment strategies.

We assume that the consumer surplus is a multiplicative function of $x$ and that it is positively affected by firms' investments (we provide a theoretical motivation for these assumptions later). Consequently, if we denote the instantaneous surplus by $S_n x$, where $n$ is the number of firms that have already invested, it holds that $S_n > S_{n-1}$ for $n \in \{1, 2\}$. To find out in which way the consumer surplus is related to the firms' investment strategies, we derive its value for each type of equilibrium.

First, if the resulting equilibrium is of the simultaneous type, the consumer surplus, $CS^S(x)$, equals

$$CS^S(x) = \frac{S_0 x}{r-\alpha} + \frac{x_1^S (S_2 - S_0)}{r-\alpha} \left( \frac{x}{x_1^S} \right)^{\beta}. \tag{19}$$

The first component in (19) equals the surplus if no investment is ever going to be made. Its second component reflects the fact that the instantaneous surplus increases from $S_0 x$ to $S_2 x$ when both firms invest at $x_1^S$.

When the resulting equilibrium is of the sequential type, the consumer surplus, $CS^L(x)$, is given by

$$CS^L(x) = \frac{S_0 x}{r-\alpha} + \frac{x_1^L (S_1 - S_0)}{r-\alpha} \left( \frac{x}{x_1^L} \right)^{\beta} + \frac{x_2^F (S_2 - S_1)}{r-\alpha} \left( \frac{x}{x_2^F} \right)^{\beta}. \tag{20}$$

Here, the consumer surplus consists of three components, which can be attributed to $i$) the current firms' operations, $ii$) the leader's investment at $x_1^L$, and $iii$) the follower's investment at $x_2^F$. The consumer surplus in the preemptive equilibrium, $CS^P(x)$, has the same form as (20), with the exception that $x_1^L$ is replaced by $\min\left(x_{21}^P, x_1^L\right)$. Comparing (19) and (20) enables us to formulate the following proposition.

**Proposition 4** *Under the preemptive/sequential equilibrium the consumer surplus is always larger than in the joint investment equilibrium.*

**Proof.** See Appendix B. ∎

In order to calculate social welfare, we need to provide a link between the consumer surplus and the firms' valuations. We do so by assuming that the firm's investments are associated with offering a product of a higher quality.[18] As long as the firms offer the same initial quality $b_0$, they compete à la Cournot. After investing first, Firm 1 achieves a Stackelberg advantage in the differentiated product market by offering higher quality $b_1$.[19] The Cournot outcome is restored in the higher quality segment after Firm 2 has invested.

We assume that a continuum of consumers exists with instantaneous utility function

$$U_i(x) = \theta_i b_k - p_k(x), \tag{21}$$

where $\theta_i$ is a consumer-specific parameter uniformly distributed over the interval $[0, \sqrt{x}]$, and $p_k(x)$ is the price of the product of quality $b_k$, $k \in \{0, 1\}$. Solving for equilibrium quantities and prices (for derivations see Appendix C) yields the following instantaneous profit functions: $\pi_{00} = \frac{b_0}{9}x$, $\pi_{01} = \frac{b_0}{16}x$, $\pi_{10} = \frac{2b_1 - b_0}{8}x$, and $\pi_{11} = \frac{b_1}{9}x$. Upon examining equation (2), it can be seen that the multipliers of $x$ can be interpreted as coefficients $D_{N_i N_j}$ in the model developed in Section 2. On the basis of equilibrium quantities and prices, also the instantaneous consumer surplus coefficients are established: $S_0 = \frac{2}{9}b_0$, $S_1 = \frac{4b_1+5b_0}{32}$, and $S_2 = \frac{2}{9}b_1$ (see Appendix C). Therefore, by assuming that consumers' preferences can be described by utility function (21), we obtain that consumer surplus is a multiplicative function of $x$ (and that it is positively affected by firms' investments).

Figure 7 illustrates the consumer surplus and the firms' valuations as the functions of cost-asymmetry parameter $\kappa$.

[Please insert Figure 7 about here]

From Figure 7 it can be concluded that a low cost asymmetry results in a relatively low consumer surplus and higher values of the firms. Raising the cost asymmetry, so that the simultaneous equilibrium is superseded by the preemptive equilibrium, leads to a downward jump in the firms' values and, at the same time, to an upward jump in the consumer surplus. As seen before, the decline in the firms' values mainly results from the need to incur the investment expenditure, $I_i$, earlier. The upward jump in the consumer surplus is the consequence of an earlier provision of the higher quality product.

When investment cost is relatively small compared to the incremental consumer surplus resulting from investment, the resulting preemptive equilibrium is socially desirable (as in Figure 7). This justifies introducing an investment subsidy for the potential first mover (the low-cost firm). Such a subsidy has two effects on investment. First, it lowers the effective investment cost, which would accelerate investment both in strategic and non-strategic situations. Second, which is a direct implication of our model, it results in the switch across equilibria so that the outcome associated with an earlier investment (preemption game) is triggered.

At this point, it is interesting to analyze whether the government could reclaim a part of the subsidy needed to trigger the switch across equilibria by auctioning it to the firms.[20] Consider a subsidy that reduces the investment expenditure from $I\kappa$ to $I$. Winning the auction by Firm $i$ is equivalent to receiving the gross payoff equal to $V_1(x) - V_2(x)$. In general, it is possible that this difference lies substantially below the subsidy level. Using the set of parameters from Section 5 with $\kappa = 1.2$, and assuming that \$1 million is the unit of analysis, we find that this difference amounts to \$10.20 million (\$62.79m − \$52.59m). Consequently, any firm will be willing to pay this amount for the subsidy equal to $I(\kappa - 1)$, i.e. \$20 million. However, the present value of such a subsidy is only \$7.09 million.[21] This means a firm would be willing to incur a net cost of \$3.11 million only to credibly commit to act as the low-cost leader in the preemption game. Obviously, if receiving the subsidy was the only way of making such a commitment, the government would profit from auctioning it.

In the opposite case of a relatively high cost, it is optimal from a welfare perspective to postpone the investment. In such a situation, an increase of $\kappa$ beyond $\kappa^{**}$, which results in a preemption game, has a detrimental effect on welfare. Corollary 5 summarizes this observation.

**Corollary 5** *There exists a critical level of investment expenditure below which social welfare is always larger in the preemptive/sequential equilibrium than in the joint investment equilibrium.*

Corollary 5 is closely associated with the impact of uncertainty on the social welfare. Other things equal, higher profit volatility discourages investment and may result in a switch from the preemptive to the joint investment equilibrium. Therefore, if entering a new market segment is associated with a low (high) investment cost, a higher level of uncertainty is likely to be associated with a socially less (more) desirable outcome.

It is important to notice that these conclusions do not carry over to the case where the first-mover advantage is large, which would occur when the product quality difference is higher. Then, as illustrated in Figure 5 the preemptive equilibrium prevails even if firms are symmetric. Consequently, from a welfare perspective introducing asymmetry may not be needed.

## 7 Conclusions

We analyze the impact of the investment cost asymmetry on the choice of duopolists' optimal investment strategies. Since the firms operate in an imperfectly competitive market, the profitability of each firm's project is affected by the other firm's decision to invest. We show that when the asymmetry between firms is relatively small and so is the first-mover advantage, the firms invest jointly. When the first-mover advantage is significant, the low-cost firm preempts the high-cost firm. In the situation where the asymmetry between firms becomes sufficiently large, the firms exercise their investment options sequentially and their mutual decisions do not affect each other directly. Moreover, we obtain that higher uncertainty delays investment not only by raising any firm's investment threshold but also by favoring equilibria associated with a relatively late investment.

Furthermore, the effects of investment cost asymmetry on the values of the two firms are explored. It is shown that the relationship between the firm's value and the cost asymmetry is non-monotonic and discontinuous. We obtain a number of counterintuitive results. For reasonable parameter values, deepening the firm's competitive disadvantage due to its marginal investment cost increase may reduce the value of its competitor. This situation results when a switch from simultaneous to preemptive equilibrium occurs upon the marginal change in the cost asymmetry. Another interesting effect of strategic interactions is present when the firms are engaged in a preemption game. Then increasing the extent to which the firm is set at a cost disadvantage leads to an appreciation of its value due to the strategic effect on the competitor's investment timing.

We also discuss the welfare effects of strategic interactions between the firms. In an example where the investment increases product quality, we show that the relationship between cost asymmetry and social welfare depends on the cost of investment. If this cost is relatively high, social welfare is maximized when none of the firms suffers from competitive disadvantage. However, if the investment cost is low, an increase of the consumer surplus that results from the early investment in the preemptive equilibrium exceeds the reduction in the firms' joint value associated with such an investment. Therefore, the preemptive equilibrium, occurring when the costs sufficiently differ, is in this case desirable. This observation allows for the conclusion that an equal access of competitors to a new technology or market segment is in some scenarios not socially optimal.

In the paper, we analyze one possible form of asymmetry. Other forms of asymmetry would include different instantaneous profit flows (due to differing unit prices, marginal costs or stochastic output flows) or, possibly, dissimilar exit options. Still, we view our results as robust, which we motivate using the following two arguments. Firstly, most of the strategic situations involving firms with asymmetric exit options or future instantaneous profit flows can be represented as the game analyzed in the paper. This is due to the possibility of incorporating the present values of such differences in the firms' investment costs. In such cases, these other sources of asymmetry would lead to *quantitatively* similar results. Secondly, we obtained a number of *qualitative* results, such as a natural emergence of the role of the leader, the switch across the three equilibria with the changing magnitude of asymmetry, the non-monotonicity of the firm values, and finally – the social welfare trade-off between early delivery of the product and committing sunk costs too early. These results are entirely driven by the relative values of the firms' investment options and the moneyness of these options.[22] It can be expected that other forms of asymmetry (thus ones that cannot be represented as a difference in the costs of investment) will only influence these two items, so that similar results will occur here. A promising extension of our framework would be to incorporate multiple sources of asymmetry, which would lead to yet a richer set of outcomes. For example, the order in which firms invest would, in general, be no longer predetermined and depend on the configuration of macroeconomic variables, such as industry growth rate and its volatility.

# A Real Options Investment Model

In this section we present the standard investment model as described in Dixit and Pindyck (1996), Ch. 6. The basic problem is to find the optimal timing of an irreversible investment, $I$, given that the value of the investment project follows a geometric Brownian motion (1). The threshold value of $x$ at which the project is undertaken maximizes the value of the firm.

For the follower, the Bellman equation in the continuation region (i.e. before the investment is made) is given by

$$rV_i^F(x)\,dt = xD_{01}dt + E\left[dV_i^F(x)\right]. \tag{A.1}$$

Equation (A.1) means that the sum of the expected capital gain and payout from the firm over interval $dt$ equals (in percentage terms) the riskless rate of return. Applying Itô's lemma to the RHS of (A.1), and dividing both sides of the equation by $dt \to 0$ results in the following ordinary differential equation:

$$rV_i^F(x) = xD_{01} + \alpha x \frac{\partial}{\partial x} V_i^F(x) + \frac{1}{2}\sigma^2 x^2 \frac{\partial^2}{\partial x^2} V_i^F(x). \tag{A.2}$$

The general solution to (A.2) has the following form:

$$V_i^F(x) = \frac{xD_{01}}{r-\alpha} + Ax^\beta + Bx^\lambda, \tag{A.3}$$

where $A$ and $B$ are constants, $\beta$ is given by (6), and $\lambda$ equals

$$\lambda = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0.$$

In order to find the value of the firm, $V_i^F(x)$, and the optimal investment threshold, $x_i^F$, the following boundary conditions are applied to (A.3):

$$
\begin{aligned}
V_i^F\left(x_i^F\right) &= \frac{x_i^F D_{11}}{r-\alpha} - I_i, && \text{(A.4)}\\
\frac{\partial}{\partial x} V_i^F\left(x_i^F\right) &= \frac{D_{11}}{r-\alpha}, && \text{(A.5)}\\
V_i^F(0) &= 0. && \text{(A.6)}
\end{aligned}
$$

Conditions (A.4) and (A.5) are called the value-matching and the smooth-pasting conditions, respectively, and ensure continuity and differentiability of the value function at the investment threshold. Condition (A.6) ensures that the firm is worthless at the absorbing barrier $x = 0$. Consequently, this condition implies that $B = 0$.

Substitution of (A.3) into (A.4)-(A.6) and some algebraic manipulation yield the value of the optimal investment threshold (5) and the value of the firm (8).

Recall that by the stopping value the value of the firm after investment is meant. The optimal investment strategy of the follower has the form of a trigger strategy since the difference between *i*) the sum of the current instantaneous payoff and the expected capital gain of the stopping value, and *ii*) the cost of holding the stopping value, i.e.

$$
xD_{01} + \frac{\alpha x D_{11}}{r-\alpha} - r\left(\frac{xD_{11}}{r-\alpha} - I_i\right), \tag{A.7}
$$

is decreasing with $x$ (cf. Dixit and Pindyck, 1996, p. 130).

The value of Firm $i$ as the leader and when the firms invest simultaneously is calculated in a similar way (i.e. by changing the instantaneous profit flow in (A.1) and modifying boundary conditions (A.4) and (A.5)).[23]

# B Proofs of Propositions

**Proof of Proposition 1.** The proof follows from the construction of preemption point $x_1^P$, at which the value of Firm 1 as the leader is equal to that as the follower, and to the right of which the value of Firm 1 as the leader exceeds its follower value. Moreover, whenever $x_{21}^P$ exists (which is true in the relevant case of the preemptive equilibrium), it holds that $x_{21}^P < x_1^F$, since $\xi_2(x)$ can be positive only in interval $\left(0, x_1^F\right)$. Finally, $x_1^P < x_1^L$ by construction of Firm 1's value functions. Consequently, inequalities $x_1^P < \min\left(x_{21}^P, x_1^L\right) < x_1^F$ imply that in the preemptive equilibrium Firm 1 extracts relative surplus from becoming the leader. ■

**Proof of Proposition 2.** The sequential equilibrium occurs when Firm 2 has no incentive to invest as the leader. Formally, this requires that $\xi_2(x)$ is negative for all $x \in [x(0), x_2^F)$. Therefore, in order to determine the domain of $\kappa$-values where the sequential equilibrium prevails, we are interested in finding a pair $(x^*; \kappa^*)$ that satisfies the following system of equations

$$\begin{cases} \xi_2(x^*; \kappa^*) = 0 \\ \left.\frac{\partial \xi_2(x^*; \kappa^*)}{\partial x}\right|_{x=x^*} = 0. \end{cases} \tag{B.1}$$

In other words, we are interested in a point $(x^*; \kappa^*)$ at which Firm 2's leader function is tangent to the follower function. After substituting (8) and (9) into (12), all defined for Firm 2 for $x \leq x_2^F$, and rearranging, (B.1) becomes:

$$\begin{cases} \frac{x^*(D_{10}-D_{01})}{r-\alpha} - I\kappa^* + \frac{x_1^F(D_{11}-D_{10})}{r-\alpha}\left(\frac{x^*}{x_1^F}\right)^\beta - \frac{x_2^F(\kappa^*)(D_{11}-D_{01})}{\beta(r-\alpha)}\left(\frac{x^*}{x_2^F(\kappa^*)}\right)^\beta = 0 \\ \frac{D_{10}-D_{01}}{r-\alpha} + \beta\frac{D_{11}-D_{10}}{r-\alpha}\left(\frac{x^*}{x_1^F}\right)^{\beta-1} - \frac{D_{11}-D_{01}}{r-\alpha}\left(\frac{x^*}{x_2^F(\kappa^*)}\right)^{\beta-1} = 0. \end{cases} \tag{B.2}$$

After multiplying both sides of the second equation in (B.2) by $\frac{x^*}{\beta}$, subtracting it from the first equation, and further rearranging, we obtain

$$x^* = \frac{\beta}{\beta-1}\frac{I\kappa^*(r-\alpha)}{D_{10}-D_{01}}. \tag{B.3}$$

Substituting (B.3) into the first equation in (B.2) and (5) for $x_1^F$ yields

$$\frac{\beta}{\beta-1}I\kappa^* - I\kappa^* + \left(\frac{D_{11}-D_{01}}{D_{10}-D_{01}}\kappa^*\right)^\beta \frac{\beta}{\beta-1}\frac{D_{11}-D_{10}}{D_{11}-D_{01}}I - \left(\frac{D_{11}-D_{01}}{D_{10}-D_{01}}\right)^\beta \frac{I\kappa^*}{\beta-1} = 0. \tag{B.4}$$

Rearranging (B.4) leads to the expression (15).

In the remaining part of the proof, we demonstrate that $\kappa^* > 1$. It holds that

$$\kappa^* > 1 \Longleftrightarrow \frac{(D_{10} - D_{01})^{\beta} - (D_{11} - D_{01})^{\beta}}{\beta\,(D_{10} - D_{11})} - (D_{11} - D_{01})^{\beta-1} > 0, \tag{B.5}$$

which can be rewritten into

$$\begin{aligned} & \frac{(D_{10} - D_{01})^{\beta} - (D_{11} - D_{01})^{\beta}}{\beta\,(D_{10} - D_{11})} - (D_{11} - D_{01})^{\beta-1} \\ = \; & \frac{(D_{10} - D_{01})^{\beta} - (D_{11} - D_{01})^{\beta} - \beta\,(D_{10} - D_{11})\,(D_{11} - D_{01})^{\beta-1}}{\beta\,(D_{10} - D_{11})} > 0. \end{aligned} \tag{B.6}$$

By substituting

$$a = D_{11} - D_{01}, \tag{B.7}$$

$$b = D_{10} - D_{01}, \tag{B.8}$$

and rearranging, we conclude that (B.6) is equivalent to

$$\frac{a^{\beta}}{\beta\,(b-a)}\left(\left(\frac{b}{a}\right)^{\beta} - 1 - \beta\frac{b}{a} + \beta\right). \tag{B.9}$$

After observing that $b > a$ and $\frac{a^{\beta}}{\beta(b-a)} > 0$, we have to prove that the second factor of (B.9) is positive. Let us denote $w = \frac{b}{a}$ and $g\,(w) = w^{\beta} - 1 - \beta w + \beta$. Consequently, we have

$$g\,(1) = 0, \text{ and} \tag{B.10}$$

$$\frac{\partial g(w)}{\partial w} = \beta w^{\beta-1} - \beta > 0, \;\forall \beta, w > 1. \tag{B.11}$$

■

**Proof of Proposition 3.** Firm 1 prefers simultaneous investment unless for some $x$ its leader payoff, $V_1^L\,(x)$, exceeds the optimal joint investment payoff, $V_1^S\,(x)$. Define $\zeta_i\,(x) \equiv V_i^L\,(x) - V_i^S\,(x)$. The simultaneous equilibrium occurs only if $\zeta_1\,(x)$ is positive for all $x \in \left(x_1^P, x_2^F\right)$. Therefore, in order to determine the domain of $\kappa$-values for which the simultaneous equilibrium prevails, we are interested in finding a pair $(x^{**};\kappa^{**})$ that satisfies the following system of equations

$$\begin{cases} \zeta_1\left(x^{**};\kappa^{**}\right)=0 \\ \frac{\partial\zeta_1(x;\kappa^{**})}{\partial x}\Big|_{x=x^{**}}=0. \end{cases} \tag{B.12}$$

In other words, we are interested in a point $(x^{**};\kappa^{**})$ in which Firm 1's simultaneous investment function is tangent to its leader function. After substituting (9) and (10) into $\zeta_1\left(x^{**};\kappa^{**}\right)$, all defined for Firm 1 for $x\leq x_1^S$, and rearranging, we obtain

$$\begin{cases} \frac{x^{**}(D_{00}-D_{10})}{r-\alpha}+I+\frac{x_1^S(D_{11}-D_{00})}{\beta(r-\alpha)}\left(\frac{x^{**}}{x_1^S}\right)^{\beta}-\frac{x_2^F(\kappa^{**})(D_{11}-D_{10})}{r-\alpha}\left(\frac{x^{**}}{x_2^F(\kappa^{**})}\right)^{\beta}=0 \\ \frac{D_{00}-D_{10}}{r-\alpha}+\frac{D_{11}-D_{00}}{r-\alpha}\left(\frac{x^{**}}{x_1^S}\right)^{\beta-1}-\beta\frac{D_{11}-D_{10}}{r-\alpha}\left(\frac{x^{**}}{x_2^F(\kappa^{**})}\right)^{\beta-1}=0. \end{cases} \tag{B.13}$$

After multiplying the second equation in (B.13) by $\frac{x^{**}}{\beta}$, subtracting it from the first equation, and rearranging, we obtain

$$x^{**}=\frac{\beta}{\beta-1}\frac{I\left(r-\alpha\right)}{D_{10}-D_{00}}. \tag{B.14}$$

Substituting (B.14) into the first equation in (B.13) and (11) and (5) for $x_1^S$ and $x_2^F$, respectively, yields

$$-\frac{I}{\beta-1}+\left(\frac{D_{11}-D_{00}}{D_{10}-D_{00}}\right)^{\beta}\frac{I}{\beta-1}-\left(\frac{D_{11}-D_{01}}{\left(D_{10}-D_{00}\right)\kappa^{**}}\right)^{\beta}\frac{\beta I\kappa^{**}}{\beta-1}\frac{D_{11}-D_{10}}{D_{11}-D_{01}}=0. \tag{B.15}$$

Given that we only consider the case that $\kappa^{**}\geq 1$, rearranging (B.15) leads to the expression (17).

In the remaining part of the proof we show that the optimality of the simultaneous investment for Firm 1 implies that Firm 2 is better off by investing simultaneously as well. Consequently, we prove that as long as it is optimal for Firm 1 to invest simultaneously, Firm 2's follower threshold is always smaller than Firm 1's optimal joint investment threshold (since if this is true, then it is always optimal for Firm 2 to invest immediately when Firm 1 invests). First, we determine $\widehat{\kappa}$ which solves

$$x_2^F\left(\widehat{\kappa}\right)=x_1^S\left(\widehat{\kappa}\right). \tag{B.16}$$

For $\kappa < \widehat{\kappa}$ it holds that $x_2^F(\widehat{\kappa}) < x_1^S(\widehat{\kappa})$. After substituting (5) for Firm 2 and (11) for Firm 1 into (B.16), and rearranging, we obtain

$$\widehat{\kappa} = \frac{D_{11} - D_{01}}{D_{11} - D_{00}}. \tag{B.17}$$

Now, we show that $\widehat{\kappa} > \kappa^{**}$, i.e. that

$$\frac{D_{11} - D_{01}}{D_{11} - D_{00}} - (D_{11} - D_{01}) \left( \frac{\beta (D_{10} - D_{11})}{(D_{10} - D_{00})^{\beta} - (D_{11} - D_{00})^{\beta}} \right)^{\frac{1}{\beta - 1}} > 0 \tag{B.18}$$

holds. After substituting

$$\begin{aligned} c &= D_{11} - D_{00}, \\ d &= D_{10} - D_{00}, \end{aligned}$$

and rearranging, we obtain that condition (B.18) is equivalent to

$$\frac{1}{c} - \left( \frac{\beta (d - c)}{d^{\beta} - c^{\beta}} \right)^{\frac{1}{\beta - 1}} > 0. \tag{B.19}$$

This implies

$$\left( \frac{d}{c} \right)^{\beta} - 1 - \beta \left( \frac{d}{c} - 1 \right) > 0.$$

Let us denote $z = \frac{d}{c}$ and $h(z) = z^{\beta} - 1 - \beta (z - 1)$. Consequently, we have

$$h(1) = 0, \text{ and} \tag{B.20}$$

$$\frac{\partial h(z)}{\partial z} = \beta z^{\beta - 1} - \beta > 0, \tag{B.21}$$

since $z > 1$ and $\beta > 1$. ■

**Proof of Proposition 4.** Since in the relevant case it holds that $x_{21}^P < x_2^F < x_1^S$, subtracting the value of consumer surplus in the joint investment equilibrium from the value corresponding to the preemptive investment yields

$$\begin{aligned} \Delta CS^{P-S}(x) &= \frac{(S_1 - S_0)\, x}{r - \alpha} \left( 1 - \left( \frac{x_{21}^P}{x_2^F} \right)^{\beta} \right) \left( \frac{x}{x_{21}^P} \right)^{\beta} \\ &+ \frac{(S_2 - S_0)\, x}{r - \alpha} \left( 1 - \left( \frac{x_2^F}{x_1^S} \right)^{\beta} \right) \left( \frac{x}{x_2^F} \right)^{\beta} > 0. \end{aligned}$$

An analogous reasoning is applied while comparing the simultaneous equilibrium with the sequential investment strategy. ■

## C Derivation of Consumer Surplus and Profit Functions

Denote the price (quantity) of the product of quality $k$ by $p_{kl}$ ($q_{kl}$), where $l$ is the quality of the product offered by the competitor. When both firms offer a product of the same quality, the resulting equilibrium is a (symmetric) Cournot equilibrium. The prices and quantities are then equal to $p_{kk} = b_k\sqrt{x}/3$ and $q_{kk} = \sqrt{x}/3$, which yields the instantaneous profit $\pi_{kk} = D_{kk}x = b_k x/9$. As it can be seen from Figure 8, the consumer surplus equals

$$S_{2k}x = \frac{1}{2}\left(b_k\sqrt{x} - \frac{b_k\sqrt{x}}{3}\right)\frac{2\sqrt{x}}{3} = \frac{2b_k}{9}x.$$

[Please insert Figure 8 about here]

After Firm 1 achieves the Stackelberg advantage by investing, the prices and quantities obtained by solving the firms' maximization problem are: $p_{10} = (2b_1 - b_0)\sqrt{x}/4$, $p_{01} = b_0\sqrt{x}/4$, $q_{10} = \sqrt{x}/2$, and $q_{01} = \sqrt{x}/4$. The instantaneous profits are therefore equal to $\pi_{10} = D_{10}x = (2b_1 - b_0)x/8$, and $\pi_{01} = D_{01}x = b_0 x/16$. This results in the consumer surplus (see Figure 9) being equal to:

$$S_1 x = \frac{1}{2}\left(b_1\sqrt{x} - \frac{2b_1 - b_0}{4}\sqrt{x}\right)\frac{\sqrt{x}}{2} + \frac{1}{2}\left(b_0\sqrt{x} - \frac{b_0}{4}\sqrt{x}\right)\frac{\sqrt{x}}{4} = \frac{4b_1 + 5b_0}{32}x.$$

[Please insert Figure 9 about here]

# Notes

[1]A discrete time analysis of a strategic options exercise is presented, among others, by Kulatilaka and Perotti (1998), and Smit and Ankum (1993).

[2]As we argue later, some of the other forms of asymmetry, such as those related to operating costs or exit options, are equivalent to the investment cost asymmetry (cf. Huisman, 2001, Ch. 8, Joaquin and Butler, 2000, and a related contribution of Ruiz-Aliseda, 2004). Yet other forms, such as differing parameters of the stochastic process, would still yield results that are qualitatively similar.

[3]The two latter equilibria are also present in Perotti and Rossetto (2000), in which the problem of cross-market entry is considered. However, in their framework it is never optimal for firms to invest simultaneously since the instantaneous profits of firms competing in both the market segments are lower than monopolistic profits realized in the firms' own market segments.

[4]Introducing asymmetry between firms together with relaxing a common assumption that firms are initially not present in the market allows us to obtain a number of outcomes, which are in fact typical for models with multiple investment opportunities (cf. Boyer et al., 2002).

[5]By imposing $D_{11} < D_{00}$ we obtain the framework of Perotti and Rossetto (2000), whereas by setting $D_{00} = D_{01} = 0$ we arrive at Huisman (2001), Ch. 8, and Joaquin and Butler (2000).

[6]If the initial realization of the stochastic process was sufficiently high, immediate investment would be optimal at least for one firm. Then mixed strategies equilibria could occur, as discussed (for identical firms) in Thijssen, Huisman and Kort (2002).

[7]Here it is important to realize that an immediate investment is in general not optimal.

[8]In case of symmetric firms an endogenous selection mechanism would have to be established to determine which firm will be the leader. Such a mechanism is mostly associated with an extension of the strategy spaces beyond the simple trigger strategies (cf. Fudenberg and Tirole, 1985, Grenadier, 2000, and Thijssen et al., 2002). Asymmetry between firms ensures that coordination is straightforward and no extension of strategy spaces is needed (cf. Joaquin and Butler, 2000, and Murto, 2003, who analyze investment and exit decisions, respectively). Our second assumption of sufficiently low $x(0)$ ensures that none of the firms invests when process (1) decreases (as in Grenadier, 1996, in which recession induced investment booms may occur).

[9]The lack of the simultaneous equilibrium in Perotti and Rossetto (2000) is exactly due to the violation of this condition.

[10]At first sight it may look surprising that the optimal threshold $x_1^L$ does not depend on the Firm 2's investment timing. This is due to the fact that the Firm 2's investment affects equally the value of Firm 1's investment opportunity and the present value of its project after the investment is made.

[11]In Boyer et al. (2002) it is the smaller firm that becomes the leader in the preemption game. Their case and ours have a common feature: the firm with a higher profitability of investment is first to invest.

[12]Threshold $x_1^L$ is affected by uncertainty via parameter $\beta$. The proof of a positive relationship between $x_{21}^P$ and $\sigma$ is quite involved and available from the authors upon request.

[13] The reason for investing at the optimal monopolistic threshold in Leahy (1993) is slightly different from that in our model. In Leahy (1993), it is just the effect of the limited upside potential of the price process that exactly offsets the dissipation of the option value of waiting to invest resulting from competition.

[14] Joaquin and Butler (2000) do not distunguish preemptive equilibria from sequential ones, whereas Huisman (2001) provides a formula analogous to (15) for the special case of a new market model.

[15]Here, the values of both firms are calculated prior to any investments being made. Therefore, in the preemptive/sequential equilibrium $V_1(x)$ is given by $\frac{xD_{00}}{r-\alpha} + \left[V_1^L(x_1) - \frac{x_1 D_{00}}{r-\alpha}\right](x/x_1)^\beta$, where $x_1$ equals $\min\left(x_{21}^P, x_1^L\right)$ in the preemptive equilibrium and $x_1^L$ in the sequential equilibrium. $V_2(x)$ is calculated using an analogous formula with $V_1^L(x_1)$ replaced by $V_2^F(x_1)$.

[16]Using a static framework, Gelman and Salop (1983) show that the profit of a smaller entrant may be positively related to its competitive disadvantage interpreted as a capacity constraint.

[17]Implications for the firms' behavior of the latter type of policies are documented by Stoneman and Diederen (1994).

[18]Another way to model the impact of investment on consumer surplus would be through capacity additions.

[19]This reflects the fact that Firm 1's comparative advantage (a higher product quality) leads to the endogenous Stackelberg leadership in a linear duopoly game when the risk dominance criterion of equilibrium selection is applied (see van Damme and Hurkens, 1999).

[20]We thank an anonymous referee for providing this idea.

[21]Recall that the present value is calculated by multiplying the actual amount received/paid in the future by the corresponding stochastic discount factor. Consequently, the present value of the subsidy received at the moments of the leader's investment, $x_{21}^P$, equals $I(\kappa-1)(x/x_{21}^P)^\beta$. In our example: $x=4$, $x_{21}^P=6.26$, and $\beta=2.32$.

[22]The moneyness of an option is defined as the ratio of the payoff from an immediate exercise and the

exercise price.

[23]To show that in the preemptive equilibrium a trigger strategy is the optimal strategy of the leader, it is necessary to incorporate a negative change in the leader's stopping value at $x_{21}^P$ due to the follower's investment, multiplied by the (infinite) hazard rate associated with this event.

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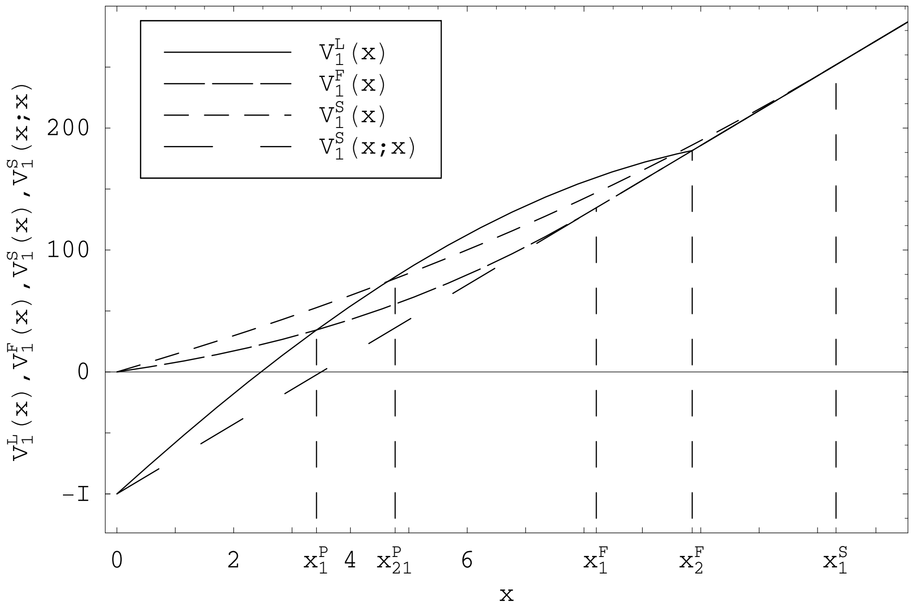


Figure 1: Firm 1's value functions when the resulting equilibrium is of the preemptive type for the set of parameter values: $r = 0.05$, $\alpha = 0.015$, $\sigma = 0.1$, $D_{00} = 0.5$, $D_{01} = 0.25$, $D_{10} = 1.5$, $D_{11} = 1$, $I = 100$, and $\kappa = 1.2$.

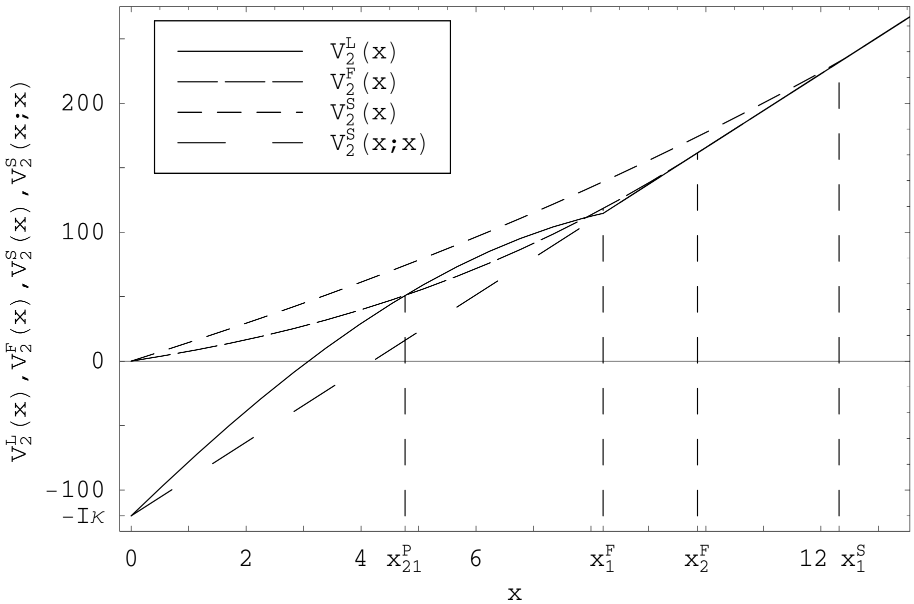


Figure 2: Firm 2's value functions when the resulting equilibrium is of the preemptive type for the set of parameter values: $r = 0.05$, $\alpha = 0.015$, $\sigma = 0.1$, $D_{00} = 0.5$, $D_{01} = 0.25$, $D_{10} = 1.5$, $D_{11} = 1$, $I = 100$, and $\kappa = 1.2$.

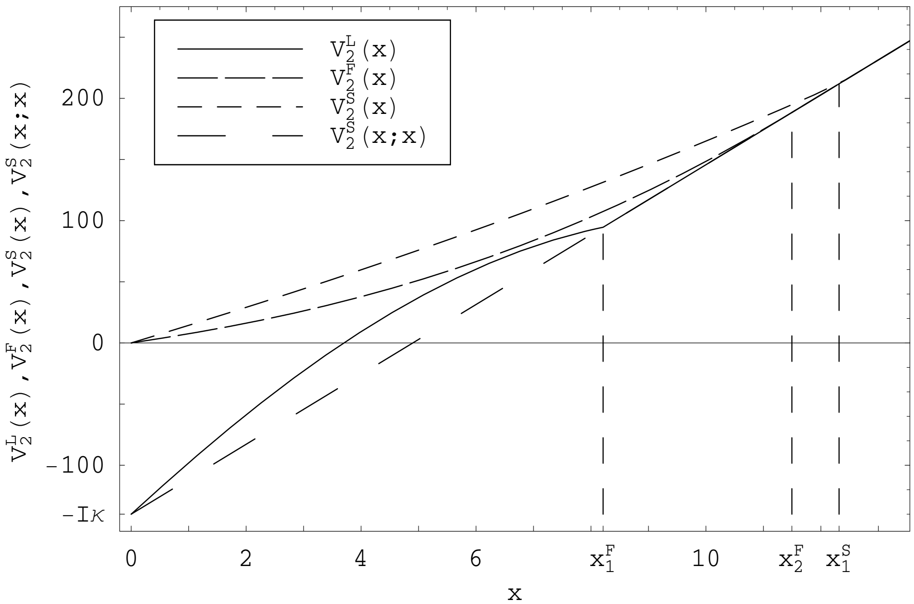


Figure 3: Firm 2's value functions when the resulting equilibrium is of the sequential type for the set of parameter values: $r = 0.05$, $\alpha = 0.015$, $\sigma = 0.1$, $D_{00} = 0.5$, $D_{01} = 0.25$, $D_{10} = 1.5$, $D_{11} = 1$, $I = 100$, and $\kappa = 1.4$.

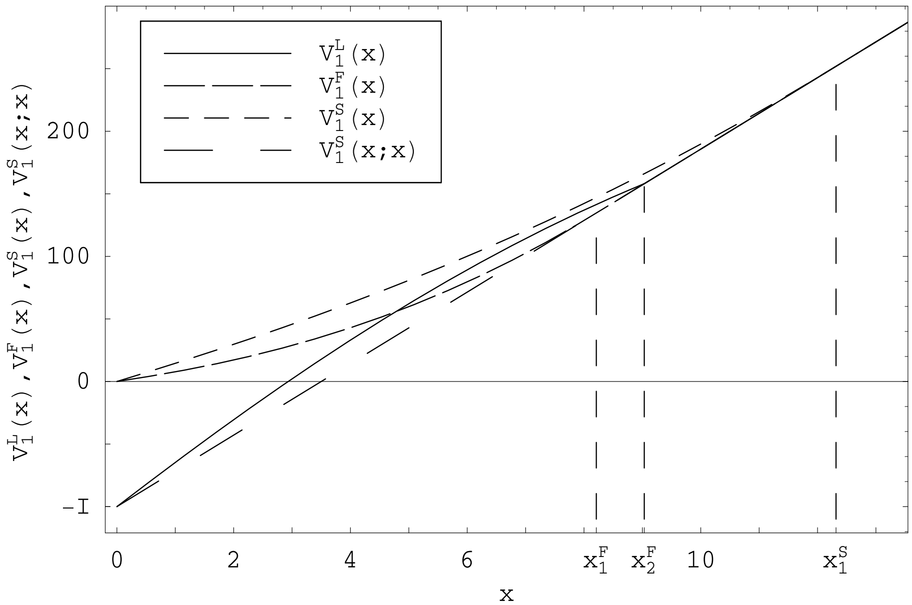


Figure 4: Firm 1's value functions when the resulting equilibrium is of the simultaneous type for the set of parameter values: $r = 0.05$, $\alpha = 0.015$, $\sigma = 0.1$, $D_{00} = 0.5$, $D_{01} = 0.25$, $D_{10} = 1.25$, $D_{11} = 1$, $I = 100$, and $\kappa = 1.1$.

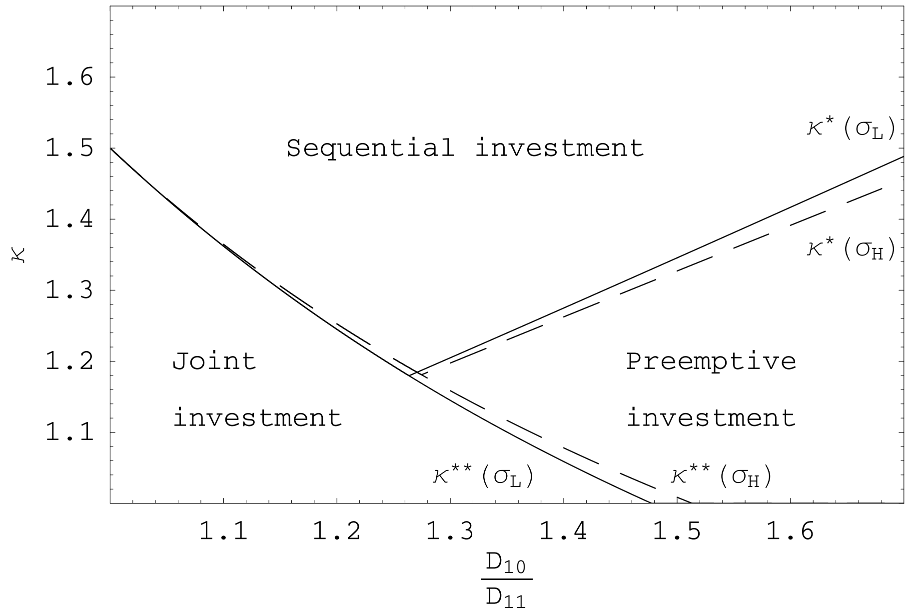


Figure 5: Regions of sequential, preemptive and joint investment equilibria for the set of parameter values: $r = 0.05$, $\alpha = 0.015$, $\sigma_L = 0.05$, $\sigma_H = 0.25$, $D_{00} = 0.5$, $D_{01} = 0.25$, and $D_{11} = 1$.

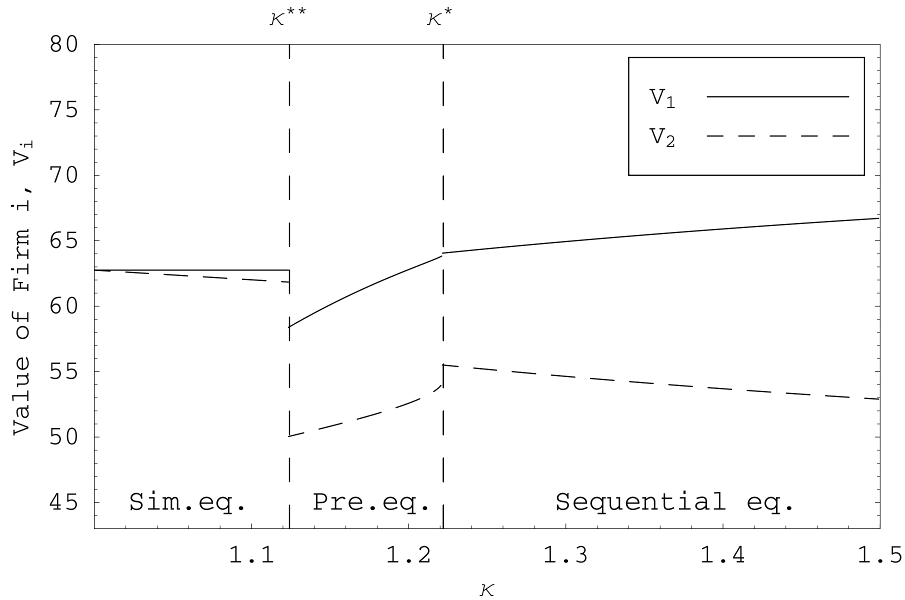


Figure 6: The value of Firm $i$ ($V_i$) corresponding to the regions of the joint investment, preemptive and sequential equilibria for the set of parameter values: $r = 0.05$, $\alpha = 0.015$, $\sigma = 0.1$, $D_{00} = 0.5$, $D_{01} = 0.25$, $D_{10} = 1.33$, $D_{11} = 1$, $I = 100$, and $x = 4$.

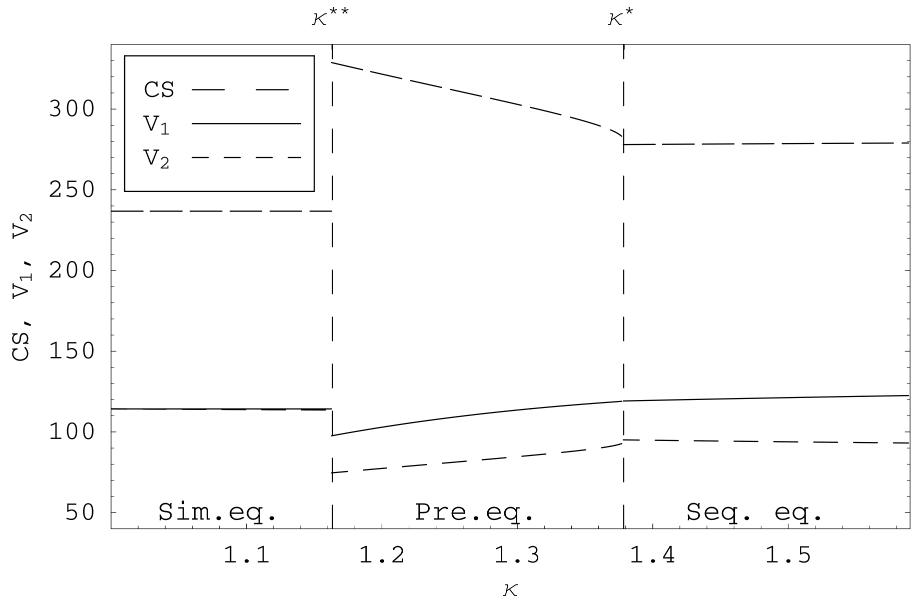


Figure 7: Value of Firm $i$ ($V_i$) and consumer surplus ($CS$) corresponding to the regions of the joint investment, preemptive and sequential equilibria for the set of parameter values: $r = 0.05$, $\alpha = 0.015$, $\sigma = 0.1$, $b_0 = 5$, $b_1 = 7$, $I = 100$, and $x = 7$.

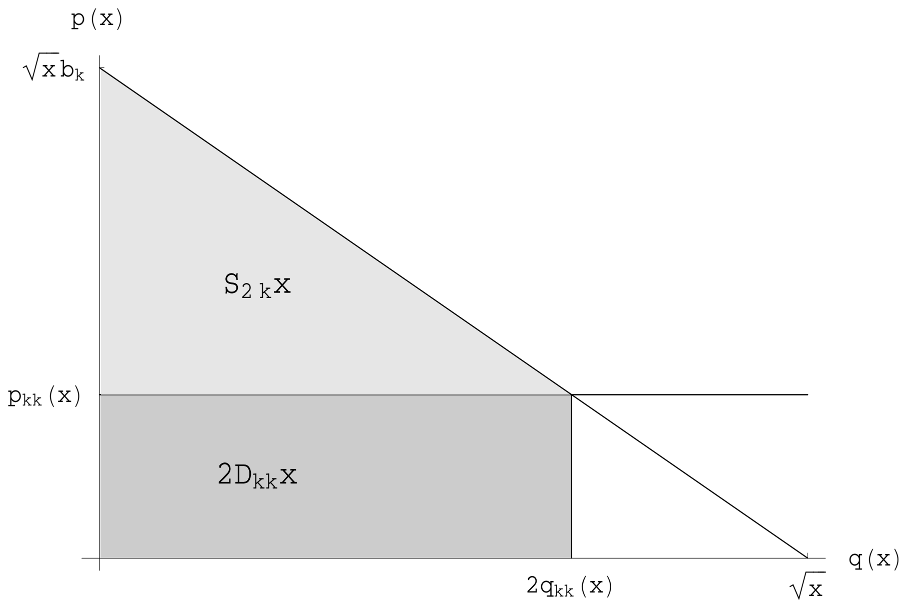


Figure 8: Firm's profits, $D_{kk}x$, and the instantaneous consumer surplus, $S_{2k}x$, in a market where firms compete with an identical product quality, $b_k$.

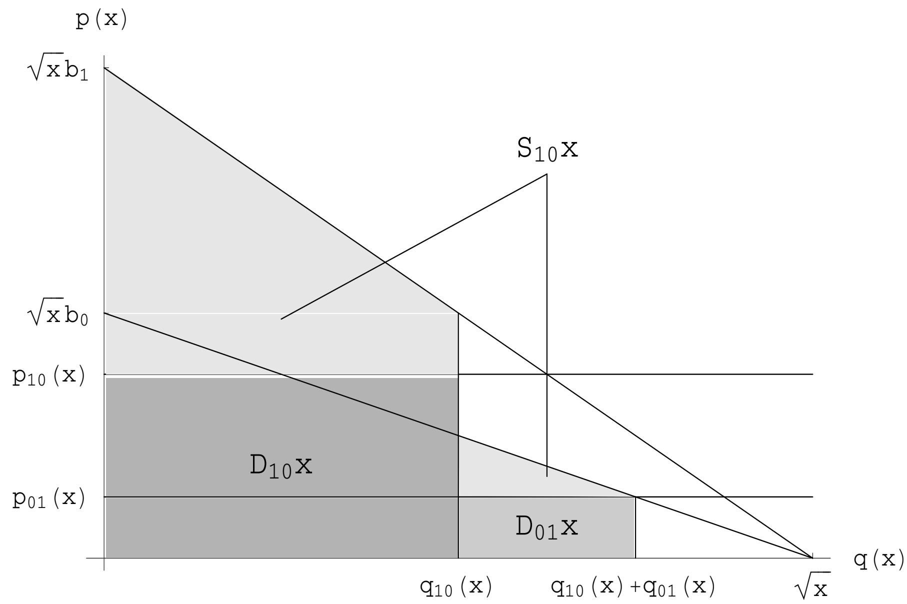


Figure 9: Firm's profits, $\pi_{10}$ and $\pi_{01}$, and the instantaneous consumer surplus, $S_{10}x$, in a market where firms compete with product qualities, respectively, $b_1$ and $b_0$.