Intrinsic concentration and generation current temperature dependence

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Temperature dependence of the current generated inside the depleted bulk under reverse bias is usually parameterised as

\[ I(T) \propto T^2 \exp(-E_{ef}/2kT) \]  \hspace{1cm} (1)

where \( T \) is the absolute temperature and \( E_{ef} \) an effective energy band gap. Often \( E_{ef} \) is set to the actual band gap, \( E_g \), which for silicon is equal to 1.12 eV at 300K. In eq. (1) the factor \( T^{1/2} \) comes from the carrier thermal velocity and the rest, obviously dominating the current temperature dependence, from the intrinsic carrier concentration \( n_i \). Thus it is crucial to know \( n_i(T) \) in detail to understand properly the \( I(T) \).

The intrinsic carrier concentration is a parameter of prime importance in semiconductor physics and a vast literature exists about it. In this Note we rely on relatively recent Review [1]. Standard temperature dependence of \( n_i \) is

\[ n_i(T) \propto T^{3/2} \exp(-E_{ef}/2kT) \]  \hspace{1cm} (2)

In this Note we concentrate at the temperature interval of \( \pm 30^\circ \text{C} \) most relevant to the present use of silicon detectors in Particle Physics. It is instructive to understand the relative weight of two temperature dependent factors in eq. (2). Calculating relative derivative \( (dn_i/n_i)/(dT/T) \) one gets

\[ (dn_i/n_i)/(dT/T) = 3/2 + E_{ef}/2kT \]  \hspace{1cm} (3)

For temperatures in the mentioned above region of interest the exponential factor dominates, e.g. for \( E_{ef} = 1.12 \text{ eV} \) and \( T = 300 \text{K} \) the second term in eq. (3) is equal to 21.7.

In Ref. [1] there are three fits of \( n_i(T) \) found in different experimental studies and described by eqs. (21, 22, 23) of this paper. They have the form of

\[ n_i(T) \propto T^m \exp(-E_a/kT) \]  \hspace{1cm} (4)

where \( E_a \) is so called activation energy. In eqs. (21, 22) \( m \) is set to standard value of 3/2 and the results can be directly compared with parameterisation (2). In eq. (22) \( m \) is
a free parameter of the fit and has the value of 2.365. At any temperature, T, the
dependence (4) can be converted to the equivalent one with \( m=3/2 \) and activation
energy \( E_a^{eq} \). Denote \( E_a^m \) the activation energy for the parameterisation (4) with
specific \( m \) and require that the relative derivative of it is equal to that for \( m =3/2 \). As a
result one gets

\[
\frac{(dn_i/n_i)}{(dT/T)} = \frac{3}{2} + E_a^{eq}/kT = m + E_a^m /kT
\]

from which it follows

\[
E_a^{eq} = E_a^m + (m-3/2) kT
\]

The activation energy in eq. (22) of Ref.[1] is \( k\times6733K = 0.580 \text{ eV} \) and \( m=2.365 \).
Using the relation (6) one obtains at \( T=273K \) the equivalent activation energy of
0.601 eV for \( m=3/2 \). Since as discussed above the contribution of \( T^m \) term to the
temperature dependence is relatively small the conversion calculation made by eq. (6)
at 0°C may be considered valid also in some temperature interval around it.

The activation energy values in eqs. (21, 22) of Ref.1 are 0.605 and 0.603 eV
respectively. Combining them with the value of 0.601 eV obtained above one gets the
average experimental value of \( E_a = 0.603 \pm 0.002 \text{ eV} \) where the uncertainty covers all
three experimental values. Therefore the experimental value for the effective gap is

\[
E_{\text{ef}} = 2E_a = 1.206 \pm 0.004 \text{ eV}.
\]

At first glance this result is incompatible with the experimental values of \( E_g \), which
according to Table 1 of Ref.1 are 1.1242 eV at 300K and 1.1367 eV at 250K.
However more thorough analysis shows that in fact there is no inconsistency.

Consider the expected temperature dependence

\[
n_i(T) \propto T^{3/2} \exp(-E_g/2kT) \tag{8}
\]

and take into account the fact that the band gap energy, \( E_g \), is also a function of
temperature. For a given temperature the relative gradient is

\[
\frac{(dn_i/n_i)}{(dT/T)} = \frac{3}{2} + E_g(T)/2kT - (dE_g/dT)/2k \tag{9}
\]

If the temperature dependent \( E_g \) is replaced in eq.(8) by a constant parameter \( E_{\text{ef}} \) then
the relative gradient will be

\[
\frac{(dn_i/n_i)}{(dT/T)} = \frac{3}{2} + E_{\text{ef}}/2kT \tag{10}
\]

Comparing eqs. (9) and (10) one gets for \( E_{\text{ef}} : \)

\[
E_{\text{ef}} = E_g(T) - T (dE_g/dT) \tag{11}
\]
Consider the situation when in some temperature interval the band gap dependence can be approximated by a linear function

\[ E_g(T) = E_0 - \alpha T. \]  
(12)

Here \( E_0 \) is the band gap value extrapolated to \( T = 0K \). In this case the effective energy gap found from eq. (11) is

\[ E_{ef} = E_0 - \alpha T - T (\cdot \alpha) = E_0 \]  
(13)

independent of \( T \) (and also of \( \alpha \)).

For temperatures above 250K, that covers our region of interest, parameterisation (12) is readily available in eq. (17) of Ref. [1]. The value of \( E_0 \) in this equation is 1.206 eV in a perfect agreement with the experimentally found \( E_{ef} \) presented in eq. (7). Thus there is no contradiction between the values of temperature dependent band gap energy and the temperature independent effective energy gap. The only surprise here is that the effective gap value required for the same temperature gradient is outside the range of the band gap values in the considered temperature interval.

Thus for non-irradiated sensors the temperature dependence of the generation current may be described by parameterisation (1) with \( E_{ef} = 1.21 \text{ eV} \). This value is based on a vast experimental material and understood theoretically. For irradiated sensors the information is much more limited. The survey [2] made in 1994 resulted in \( E_{ef} = 1.24 \pm 0.06 \text{ eV} \), which agrees with the above value for non-irradiated sensors but has a substantial uncertainty. Until more accurate data for irradiated sensors are available it looks reasonable to use \( E_{ef} = 1.21 \text{ eV} \) for irradiated sensors as well.

In conclusion, the temperature dependence of the generation current should be described by parameterisation (1) with \( E_{ef} = 1.21 \text{ eV} \) both for non-irradiated and irradiated sensors. The difference between the actual and effective energy gaps is due to using temperature independent \( E_{ef} \) instead of the temperature dependent \( E_g \).

References