ITP Domain 1

Challenges with Bearings Only Tracking for Missile Guidance Systems and How to Cope with Them

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Abstract: *This paper addresses the problem of closed loop missile guidance using bearings and target angular extent information. Comparison is performed between particle filtering methods and derivative free methods. The extent information characterizes target size and we show how this can help compensate for observability problems. We demonstrate that exploiting angular extent information improves filter estimation accuracy. The performance of the filters has been studied over a testing scenario with a static target, with respect to accuracy, sensitivity to perturbations in initial conditions and in different seeker modes (active, passive and semi-active).*

Keywords: closed loop guidance systems, bearings only tracking, object extent, robustness, particle filtering, derivative free estimators, square root unscented Kalman filter.

1. Introduction

In recent years there has been significant interest in sequential Monte Carlo methods (particle filters) [1,2] and derivative-free filters [3] applied to nonlinear estimation problems, including positioning, localisation, [4,5] and estimation for guidance systems, e.g., [6,7]. However, with a few exceptions [8,9], the applications are essentially in open-loop. Here our focus is on closed loop missile homing guidance problems.

For missile homing guidance, an estimator is required to estimate the relative motion of the target with respect to the missile. Traditionally Kalman Filters, or Extended Kalman Filters (EKFs) [3], are used. In addition guidance system models have inherent nonlinearities and non Gaussian system and/ or measurement noises which require more advanced techniques. Seeker measurements, for instance, can have significant nonlinearities and the estimation and control units have to respond quickly to target manoeuvres. In addition what a missile sees affects what it does, and what it does affects what it sees. Thus any 'parasitic' errors within the system cause missile body-motion to corrupt seeker measurements. In this situation it is not the open loop accuracy of the estimation that is of prime importance, but how errors build up. Hence, closed-loop stability becomes important. Generally, this limits the innovation gains and associated bandwidth of conventional estimators. A further restriction on bandwidth is that noise propagation onto the missile control surfaces must be kept within bounds. Only when the filter model is well matched to the 'real-world' dynamics/measurements good estimation may be obtained using low filter gains (or their equivalent).

This work investigates the performance of particle filters in the context of closed loop environment, compared with derivative free methods and their sensitivity to perturbations on the initial conditions. One of the first derivative free techniques for guidance systems was developed by MBDA in the late 80's [10]. Now there exists a wide variety of similar methods aimed at improving the estimation accuracy. Different approaches to the derivation of these filters exist such as by the unscented transformation leading to the Square Root Unscented Kalman Filter (SRUKF) [11,12] (an improved version of the Unscented Kalman Filter (UKF) [13,14]) or by divided-differences of first or higher order [15]. Here the SRUKF will be used.

The problem considered is strap-down sightline-rate estimation for missile homing guidance. The missile carries a seeker and inertial instruments fixed to the missile body (gyros and accelerometers) that in effect allow it to measure the angular bearing of a target. It is also assumed that onboard signal processing allows the angular extent of some fixed feature on the target to be measured. This is useful additional information, particularly if, as here, the problem is restricted to that of homing onto a fixed asset. Without extent information the problem is a bearings only problem. Bearings only tracking has been a problem widely studied in the literature in different contexts [16] but mainly for open loop systems, with some exceptions, e.g., [17].

The remaining part of this paper is organised as follows. Section 2 presents the state equations for the missile guidance system and the measurement model. The proposed approximate solutions to the filtering problem for closed loop guidance are outlined in Section 3. Results are presented in Section 4, and Section 5 summarises the conclusion and future work.

2. System Dynamics and Observation Models

The state vector components are chosen as follows:

$$x = \left(\psi_s - \psi_b, \dot{\psi}_s, \frac{1}{R}, \frac{v_c}{R}, \frac{d}{R} \right)', \qquad (1)$$

where ψ_s is the sightline angle, ψ_b is the missile body angle, $\dot{\psi}_s$ is the sightline rate, R is the relative distance of the missile with respect to the target, v_c is the closing speed, and d is a fixed distance. Figure 1 shows the angles in the engagement geometry.The ratio *(d/R)* represents the angular extent of some feature on the target (*d* is assumed to be 10 metres in the considered testing scenario). Figure 2 illustrates this. The system dynamics are described by the equations:

$$\begin{cases} d(\psi_s - \psi_b)/dt = \dot{\psi}_s - \dot{\psi}_b \\ d\dot{\psi}_s/dt = 2(v_c/R)\dot{\psi}_s - a_m \cos(\psi_s - \psi_b)(1/R) \\ d(1/R)/dt = (1/R)(v_c/R) \\ d(v_c/R)/dt = (v_c/R)^2 + a_m \sin(\psi_s - \psi_b)(1/R) \\ d(d/R)/dt = (v_c/R)(d/R) \end{cases} \qquad (2)$$

A polar coordinate system has been used which tends to prevent sightline rate estimates needed for guidance being corrupted when range is uncertain or unobservable. However, this results in non-linearity in the equations. Notice that $(v_c/R) \approx (1/t_{go})$ where t_{go} is the time-to-go. A digital approximation to the state equations over a sample time T (here 0.01 sec) is used within the filtering techniques. The quantities $\dot{\psi}_b T$ and $a_m T$ are provided by the gyro and accelerometer (here assumed to be accurate), where a_m is the missile acceleration.

The seeker measurements are defined by the vector $z_k \in \Re^4$ where

$$\begin{cases} z_{1,k} = \eta + (R/R_{ref})^{stype} v_{1,k} \\ z_{2,k} = R + v_{2,k} \\ z_{3,k} = v_{c,k} + v_{3,k} \\ z_{4,k} = (d/R) + v_{4,k} \end{cases} \qquad (3)$$

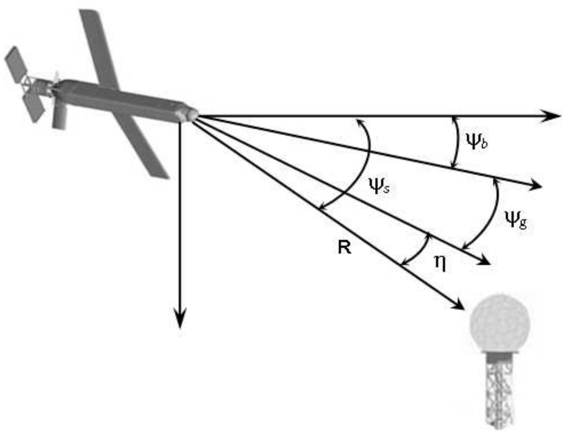

Figure 1. Definition of angles for sightline

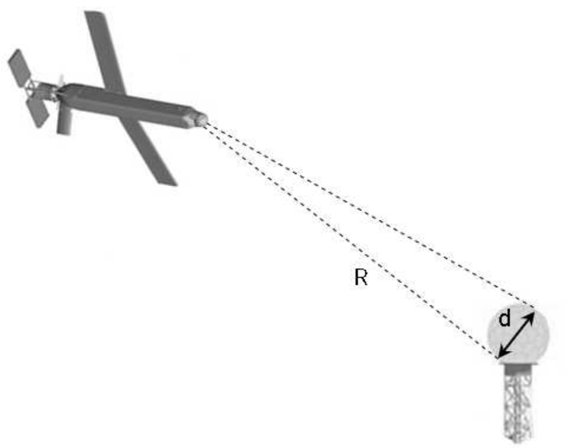

Figure 2. Angular Extent Definition

The primary seeker measurement is of boresight error $\eta = (\psi_s - \psi_b) - \psi_g$, where $(R/R_{ref})^{stype} v_{1,k}$ is a range dependent boresight error measurement noise dependent upon whether the seeker type (stype) is passive, semi-active or active, with respective values (0,1,2) and $v_{x,k}$ is vector of noise inputs such that $v_{1,k} \in \Re^4$ and $v_k \sim N(0, \Sigma_{v_k})$ where Σ_{v_k} is the measurement noise covariance.

The measurement equations expressed in terms of the chosen system states are:

$$\begin{cases} \hat{z}_{1,k} = (\psi_s - \psi_b) - \psi_g \\ \hat{z}_{2,k} = (1/R)^{-1} \\ \hat{z}_{3,k} = (v_c/R)(1/R)^{-1} \\ \hat{z}_{4,k} = (d/R) \end{cases} \qquad (4)$$

where ψ_g is the seeker gimbal angle, here assumed accurately determined at each measurement time.

In this study two cases are considered, where the measurement vector is chosen in the following forms:

- ***H*** = (1; 0; 0; 0) which corresponds to a boresight error measurement only
- ***H*** = (1; 0; 0; 1) corresponding to boresight error and an angular extent measurement *d/R*.

The extent characterises the size of the object and is provided by imagery or radar data.

Bearings only tracking with bearing only measurements is a difficult filtering problem due to lack of observability. For any given bearing to a target there is an infinitude number of target trajectories that correspond to observed bearing. The observability issue therefore is whether there exists a bijection between observation and system state – that is whether there is unique solution to the filtering problem. Figures 3 and 4 illustrate non-unique solutions for a maneuvering target with known initial range, and for a constant velocity target with unknown initial position respectively. Observability gramians are admissible mechanisms for determining observability for stochastic linear stationary systems and an extension of this can be applied to stochastic linear non-stationary systems. However, there is little in the literature addressing closed-loop guidance scenarios for stochastic non-linear non-stationary systems.

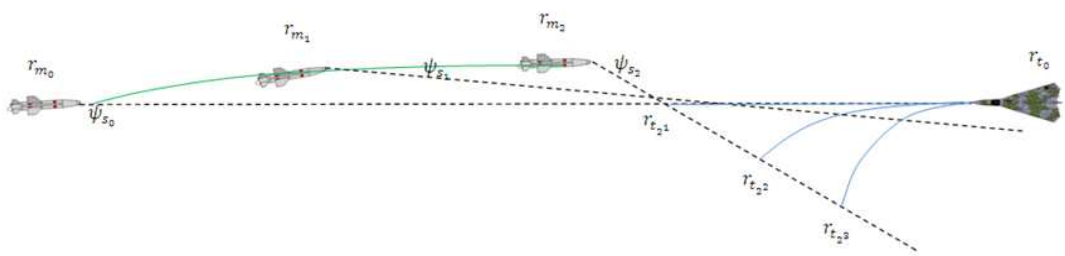

Figure 3. Given the bearings only measurement ψ_{s_0}, ψ_{s_1}, ψ_{s_2} at times t_0, t_1, t_2, respectively, a number of possible trajectories are illustrated for a manoeuvring target. Here the initial target position is known *a priori.* The missile trajectory is shown in green and 3 possible target trajectories in blue. The missile position at times t_0, t_1, t_2 are indicated by $r_{m_0}, r_{m_1}, r_{m_2}$ and target position at times t_0, t_2 denoted respectively by r_{t_0}, r_{t_2}.

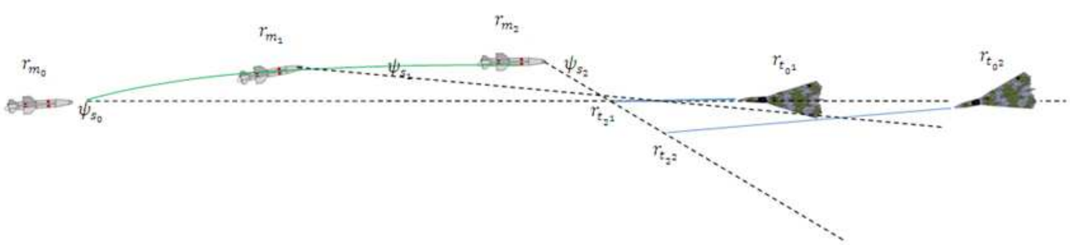

Figure 4. A constant velocity target manoeuvre without range information, target velocity and target heading has a multiplicity of viable target manoeuvre hypotheses (of which only 2 are shown). Here the initial target position is not well known.

3. The Developed Filters for Guidance Systems

3.1. A Particle Filter

The strength of particle filters [1-4] lies in their ability to represent multi-modal non-Gaussian posterior density functions by a finite set of point masses, or particles. Within the particle filtering framework a finite set of point masses is propagated through the system state and measurement equations rather than approximation of these functions based on direct linearisation (as it is done in the EKFs). The way in which the point masses are selected and how the weights are ascribed are the principal difference between the algorithms in this class of filter. The main steps of this approach are *prediction* and *update* of the samples, followed by a *resampling* stage aimed at introducing diversity in the samples.

Propagation of particles through the true system dynamics does not require the linearisation of the state equations. In the Extended Kalman Filter this entails forming a truncated, normally first or second order, Taylor Series expansion requiring the explicit calculation of Jacobians. This process is error prone, and somewhat tedious. The second order truncation also introduces quantifiable errors in the analytical propagation of the system dynamics. The particle filtering methods presented here are all derivative-free estimators by their nature.

These methods also have the advantage of being free from linearity or Gaussianity model constraints. It is interesting to note that sigma points filters (the Square Root Unscented Kalman Filter (SRUKF) [11,12], Divided Difference Filter (SRDDF) [15] and Singular Value Decomposition Kalman Filter (SVDKF) [11]) all select the regression points deterministically in contrast to stochastically generated particles Monte Carlo methods.

3.2. The Square Root Unscented Kalman Filter

The Square Root Unscented Kalman Filter (SRUKF) [11,12] propagates the square root S_k of the *a posteriori* estimate covariance matrix rather than the full covariance matrix, where $P_k = S_k S_k^T$. The algorithm uses the unscented transform to propagate the first and second order statistics of deterministically chosen points called *sigma points* through the system and measurement model.

A QR decomposition and Cholesky update replaces the weighted sums in the one step prediction and measurement update equations in the UKF for the calculation of the *a priori* state estimation error covariance $S_{x_k}^- = \sqrt{P_{x_k}^-}$ and the predicted measurement error covariance $S_{z_k}^- = \sqrt{P_{z_k}^-}$. The cross variance $P_{x_k z_k}$ is still calculated by a weighted sum of the transformed sigma points, $\hat{\chi}_{k|,k-1,i}$ and $\hat{Z}_{k|,k-1,i}$, respectively, as follows

$$P_{x_k z_k} = \sum_{i=0}^{2N} w_i^c \left(\hat{\chi}_{k|k-1,i} - \hat{x}_k^-\right)\left(Z_{k|k-1,i} - \hat{z}_k^-\right)^T, \quad (5)$$

where w_i^c are the weights, $\hat{x}_k^-$ is the predicted state estimate and $\hat{z}_k^-$ predicted measurement.

Orthogonal factorisation is asymptotically upper bound by $O(n^3)$, thus for state estimation the computational complexity of the algorithm is unchanged. However, there are benefits derived from the numerical stability square root form, such as the assured positive semi-definite nature of the matrix S_k and the reduced dynamic range.

This is followed by a rank one Cholesky update of the factor $S_k \in R^{nxn}$ with a column vector $\chi_i \in \Re^{nx1}$) which is asymptotically upper bound by $O(n^2)$. The covariance matrix $P_k = S_k S_k^T$ is updated by χ_i where $S_k = \sqrt{P_k}$, and $P_k^{'} = P_k \pm \sqrt{W_i} \chi_i \chi_i^T$ and is represented by the function by *cholupdate*(S_k , χ_i , W_i).

The Kalman Gain is calculated by

$$K_k = P_{x_k z_k} P_{z_k z_k}^{-1} = P_{x_k z_k} \left(S_{z_k}^{-\,T} S_{z_k}^-\right)^{-1}. \quad (6)$$

Respectively the state estimate vector $\hat{x}_k$ is calculated from the equation:

$$\hat{x}_k = \hat{x}_k^- + K_k\left(z_k - \hat{z}_k^-\right). \quad (7)$$

4. Testing Scenario and Evaluation

The simulated engagement geometry is shown on Figure 5. The seeker type in these simulations is passive with no range dependency in the boresight error noise. The filters were initialised with an initial condition perturbation for range and range rate by adding a zero mean random noise with a standard deviation of 5% from truth values. The simulation parameters are detailed below. The target is static.
The following initial conditions are chosen in the experiments: missile position [0, 0] *m*,

missile velocity [700, 0] ms^{-1}, target position [4666, 0] m, and target velocity [0, 0] ms^{-1}.

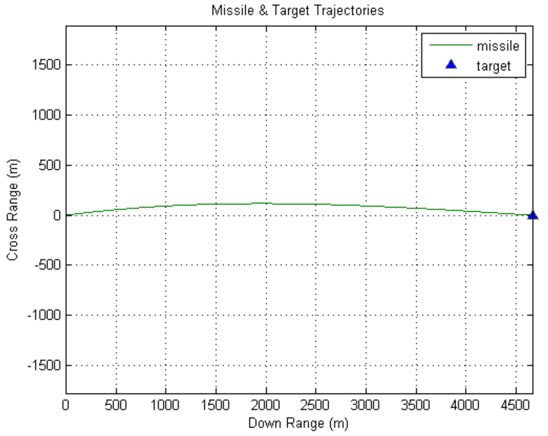

Figure 5. Considered engagement geometry with static target

The process noise covariance matrix is: $\boldsymbol{Q}$ = diag{10^{-8} rad^2, $6.26\text{x}10^{-8}$ $(rad/s)^2$, $1\text{x}10^{-8}$ m^{-2}, $1.5\text{x}10^{-7}$ s^{-2}, $1\text{x}10^{-7}$} and the measurement noise covariance matrix is: $\boldsymbol{R}$ = diag{10^{-6} rad^2, $25m^2$, 4 $(\mathbf{m/s})^2$, 10^{-6}}. The derivative-free filters (UKF and SRUKF) are initialised with the following covariance matrix: $\boldsymbol{P}_0$ = diag{10^{-4} rad^2, $4\text{x}10^{-4}$ $(rad/s)^2$, $1\text{x}10^{-8}$ m^{-2}, $4\text{x}10^{-4}$ s^{-2}, $1\text{x}10^{-6}$ }. The particle filter was run with 10000 particles and with the multinomial resampling scheme. There are 10% perturbation in the initial state estimates. A comparison was done with and a SRUKF with parameters are $\alpha = 0.445$, $\beta = 2$, $\kappa = 0$, $(\gamma) = 1$.

4.1. Comparative Analysis of Filter Performance

Figures 6-9 show the Root Mean Square Error (RMSE) between the estimated state value and the true values for boresight error, sightline rate, time-to-go and range. The experiments are performed over 200 independent Monte Carlo runs.

In respect of boresight error estimation with bearings only measurements (B) the PF performs a little better than the SRUKF. The addition of extent information (bearing and extent, BE) improves observability in both cases, delaying the final onset of divergence due to time-to-go uncertainty. The PF again outperforms the SRUKF. Similar conclusions apply based on sightline rate estimation.

Results for time-to-go estimation show that the PF (BE) starts to converge towards the end of the engagement. Early on noise tends to swamp the extent measurement. The PF (B) diverges as one might expect because range and speed are not observable. Results for the other filters are misleading because the initial perturbation in range that were used are too small to trigger strong divergence, but nor is there any evidence of convergence. In the case of the particle filter, although the initial particle cloud mean is only slightly perturbed, there is nevertheless a much wider distribution of values within the cloud that leads to initial divergence.

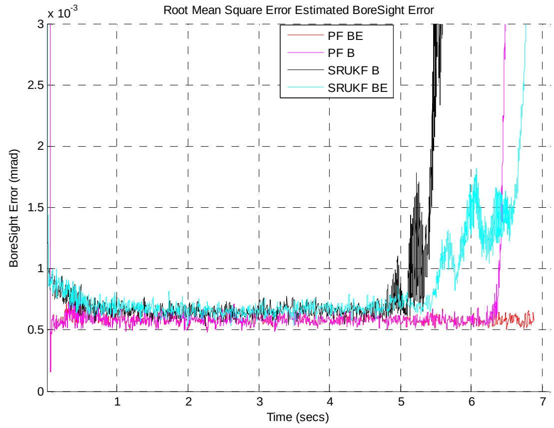

Figure 6. RMSE for boresight error estimates for the PF & SRUKF *i*) bearings only (B), *ii*) bearings & extent (BE)

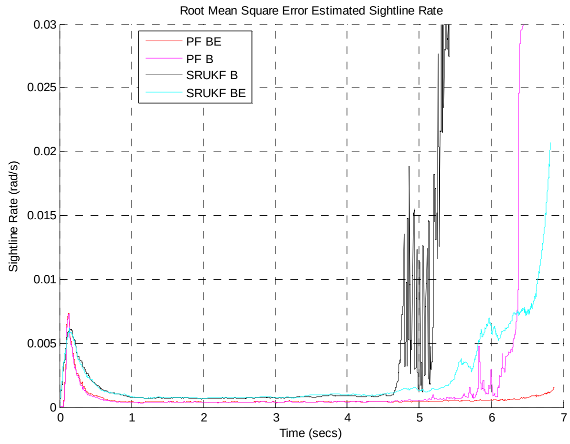

Figure 7. RMSE for sightline rate estimates for PF & SRUKF: *i*) bearings only (B), *ii*) bearings & extent (BE)

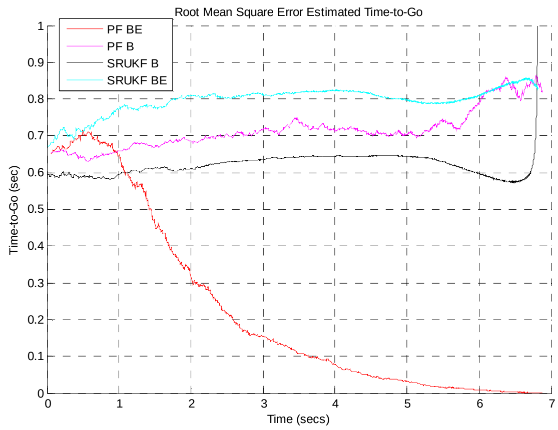

Figure 8. RMSE for time-to-go estimates (PF & SRUKF)

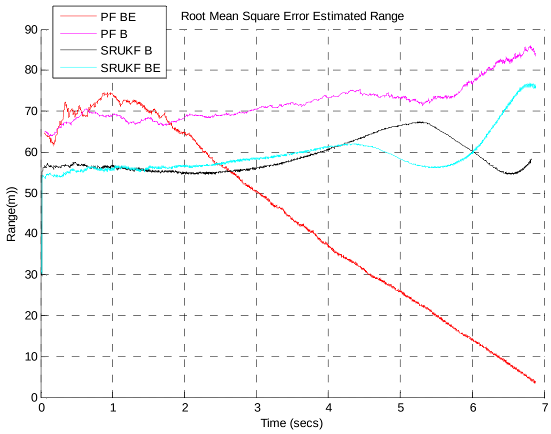

Figure 9. RMSE for range estimates for the PF and SRUKF

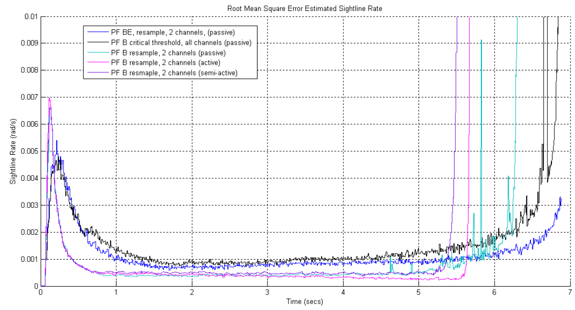

Figure 11. RMSE for sightline rate for passive, semi-active and active bearings only homing

4.2. Passive, Semi-active and Active Homing

For the purposes of guidance systems, estimators should be able to work equally well in different modes, such as passive, active and semi-active, and if necessary respective adaptations of filter parameters accomplished.

Using the same scenario and parameters, 200 independent Monte Carlo simulations were run for each seeker type. A number of combinations of filter parameters were used. The RMSEs are shown on Figures 10-12. Respective state PF estimates with the actual states are given on Figures 13-15 for different sensing modes. The PF filter performance in semi-active and active regime is better than in passive regime.

Figures 16 - 18 show results from the SRUKF with bearings only for passive, semi-active and active seeker types respectively. The highest estimation accuracy is achieved in the active seeker mode. This can be explained with the smaller measurement errors and better range determination compared with the other seeker's modes.

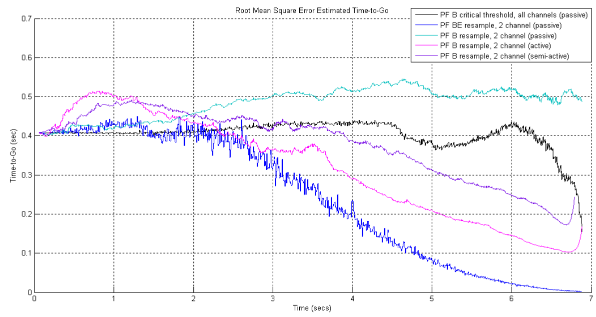

Figure 12. RMSE for time-to-go for passive, semi-active and active bearings only homing

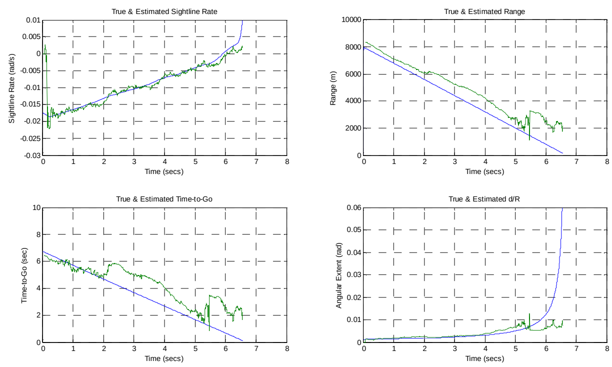

Figure 13. **Passive Seeker**. Particle filter state estimates from a single run, the following parameters: 5% perturbations of initial conditions Q= diag{1e-7 rad^2, 6.26x10^{-8} (rad/s)^2, 1e-11, m^{-2}, 1.5x10^{-7} s^{-2}, 1x10^{-7} rad^2}.

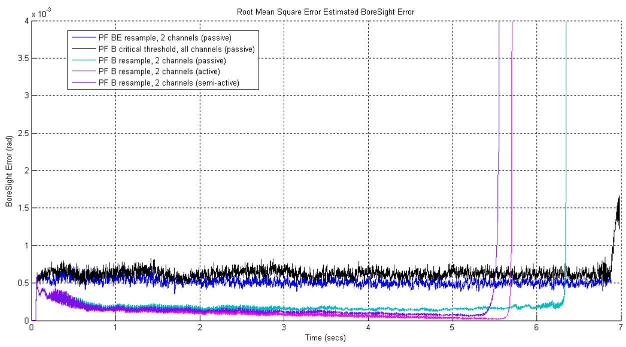

Figure 10. RMSE for boresight for passive, semi-active and active bearings only homing.

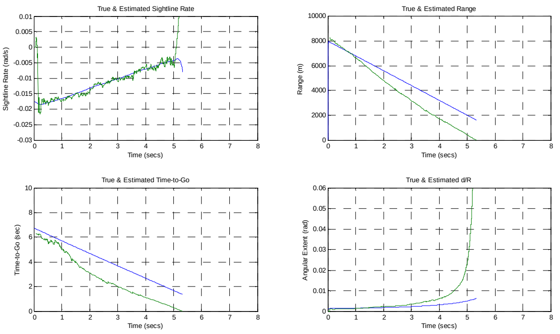

Figure 14. **Semi-active Seeker. Particle Filter.** State estimates from a single run, the following parameters: 5% perturbations of initial conditions Q= diag{1e-7 rad^2, 6.26x10^{-8} (rad/s)^2, 1e-11, m^{-2}, 1.5x10^{-7} s^{-2}, 1x10^{-7} rad^2}.

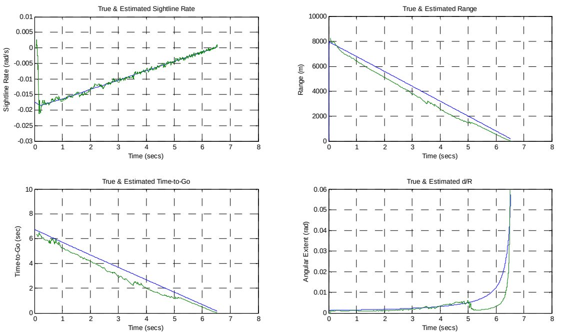

Figure 15. **Active Seeker Particle Filter.** State estimates from a single run, the following parameters: 5% perturbations of initial Q= diag{1e-7 rad^2, 6.26x10^{-8} (rad/s)^2, 1e-11, m^{-2}, 1.5x10^{-7} s^{-2}, 1x10^{-7} rad^2}.

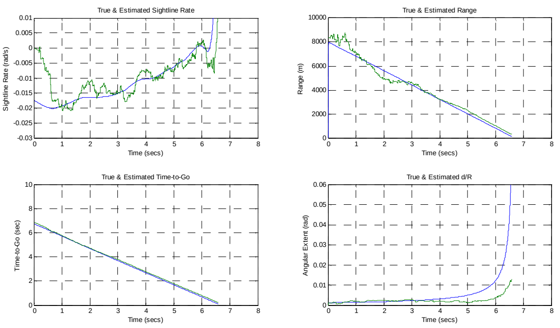

Figure 16. **Passive Seeker SRUKF** State estimates from a single run, the following parameters: 5% perturbations of initial conditions Q= diag{1e-^{8} rad^2, 5x10^{-7} (rad/s)^2, 1e-11, m^{-2}, 1x10^{-7} s^{-2}, 1x10^{-8} rad^2}.

4.3. Sensitivity to Changes in the Initial Conditions

Studies have been performed with respect to different perturbations in the initial state estimates and about the sensitivities of the filters to these conditions. All filters can cope with small initial condition changes (up to 10-15%). However, perturbations, above 20% are significant and can lead to erroneous results. In such cases the particle filter performance can be improved if the whole cloud of samples is moved towards more likely regions, with optimisation procedures, such as those described in [14].

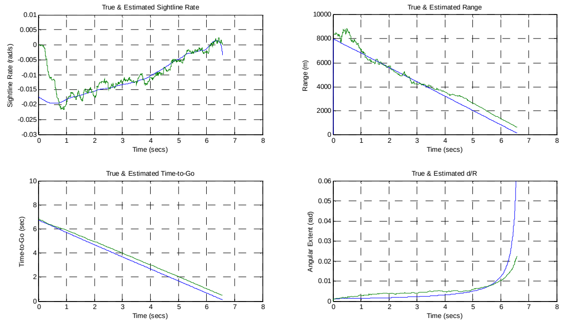

Figure 17. **Semi-active Seeker SRUKF.** State estimates from a single run, the following parameters: 5% perturbations of initial conditions Q= diag{1e-^{8} rad^2, 5x10^{-7} (rad/s)^2, 1e-11, m^{-2}, 1x10^{-7} s^{-2}, 1x10^{-8} rad^2}.

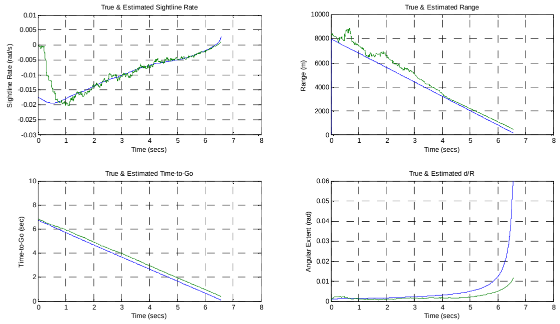

Figure 18. Active Seeker SRUKF State estimates from a single run, the following parameters: 5% perturbations of initial conditions Q= diag{1e-^{8} rad^2, 5x10^{-7} (rad/s)^2, 1e-11, m^{-2}, 1x10^{-7} s^{-2}, 1x10^{-8} rad^2}.

4.4. Open Issues for Future Research

The development of particle filtering techniques as part of guidance systems for manoeuvring targets poses different challenges and is another area under investigation. One single model is not capable of describing all modes/ regimes of the manoeuvring targets. Then multiple model particle techniques are the potential solutions in these cases.

Also stochastic estimators, equipped with automatic detection schemes will be developed so that the missile guidance system can automatically detect the objects in different environments.

5. Conclusions

In this paper a comparison of the performance of a particle filter with derivative free estimators is performed for static targets, with two types of measurements: *i*) bearings only and *ii*) bearings with extent data (for the object size). Adding extent information not only improves the accuracy but helps coping with the lack of observability. Studies have been performed also with constant velocity targets and the results in terms of accuracy are similar to those with static targets. The experiments show that boresight error and sightline rate estimation are comparable for PF and SRUKF.

In the final, non escape zone (the last 50-100 meters) the error of all algorithms is increased due to several factors, including lack of observability.

Other conclusions are:

- adding more measurements such as range rate does not improve accuracy significantly
- adding extent information to bearings measurements helps remove sightline rate divergence at close range.
- in terms of accuracy the PF and SRUKF results are comparable
- short range divergence may also be mitigated in active homing where range dependent noise correlation is exists

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