A Fuzzy Approach to Building Thermal Systems Optimization

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Abstract
Optimization of building thermal systems is treated in the paper in the framework of fuzzy mathematical programming. This new approach allows to formulate more precisely the problem which compromises energy saving and thermal comfort satisfaction under given constraints. Fuzzy optimization problem is solved analytically under some assumptions. An example illustrates the viability of the approach proposed. A solution which significantly (with 38%) improves comfort is found which is more energetically expensive with only 0.6%.

1. Introduction
In the last two decades there is an increasing interest in the optimum design and control of thermal building systems [4]-[17]. Besides of economical efficiency it has ecological aspects lowering the "green house" effect, fuel consumption and hence reducing emission of pollutants while keeping thermal comfort of the occupants [4]-[7]. The objective is to design and control building thermal system such that to minimize the capital as well as operational costs (mainly energy consumption) while satisfying all constraints as well as thermal comfort requirements. Solution of this problem is usually based on random search or gradient-based techniques. In last few years successful application of GA to this problem has been reported [5].

From the other hand, fuzzy sets have been intensively applied to building thermal behavior modeling and control [8]-[17] as more robust and easy designed. Optimization of such fuzzy models and controllers, however, has not been reported.

A general formulation of the optimization problem of a thermal plant design and control is proposed in present paper. The objective is twofold: minimization of energy consumption and maximization of the comfort. Due to the symmetrical treatment of objectives and constraints in fuzzy optimization this multiobjective problem could be transformed to non-fuzzy (crisp, conventional) optimization problem [1-2], that is

\[ J_1 = \text{Costs (variables)} \rightarrow \min \]
\[ J_2 = \text{Comfort (variables)} \rightarrow \max \]
\[ \text{s.t. Constrained (variables)} = 0 \]

Formulation of some of constraints with fuzzy sets seems more realistic and allows a higher flexibility to describe better the really existing uncertainty. For example, fans are used with a part of their design load range: the loads lower than given thresholds are not so effective. A parameter (partial load ratio, PLR) represents the load used as...
a ratio to the designed load. Practically, loads lower than a given threshold (let say, \( \text{PLR}_{\text{critical}} = 0.2DL \); where DL denotes the design load) are not used (line 1 on the Fig.1). A fuzzy constraint over the fan’s load is more realistic (line 2 on Fig.1). According to this fuzzy constraint, PLR in a range of \([0.15; 0.25]\) are acceptable, but with degree less than 1.

Practically, the optimization problem is solved as an uniobjective one where comfort requirements are considered as a constraint [4-7]

\[
J = \text{Costs (variables)} \rightarrow \min \quad (2)
\]

\[
s.t. \quad | \text{PPD (variables)} | \leq 10\%
\]

Constraints (variables)=0

where PPD denotes predicted percentage of dissatisfied people

This simplification makes the problem a rough approximation of the real intention of the designer. It is so, if conventional technique is applied, because it simplifies the solution of the problem. Fuzzy sets offer a tool for coming to grip with this problem. Comfort requirements could be formulated with an appropriate, subjectively specified membership function (Fig.2).

The problem of thermal building system became fuzzy multiobjective optimization one:

\[
\text{Costs (variables)} \rightarrow \min \quad \mu_{\text{costs}} \quad (3)
\]

\[
\text{Comfort (variables)} \rightarrow \max \quad \mu_{\text{comfort}}
\]

\[
s.t. \quad \text{Constraints (variables)} = 0 \quad \mu_{\text{constraints}}
\]

where ~ denotes fuzzy

Part of constraints could be fuzzy and other one - crisp. Building model (which could be of the fuzzy rule-base type) as well as models of components of the thermal system are not considered due to simplicity.

Fig.1 Fan’s partial load ratio constraint

1-conventional case
2-fuzzy (flexible) constraints

Fig.2 Fuzzy comfort membership functions
1 - conventional case
2 - very strong; preference of slightly cool environment
3 - slight violation
4 - if possible in the range \([-10; 10]\) %

3 Optimization problem solving

Solution of the optimization problem (3) could be found numerically [1-2]. For some special cases it could be found analytically [3]. We consider a simple example of a thermal building system optimization in order to illustrate the approach proposed.

Suppose that cumulative costs (capital and running) depends on PPD as follows (Fig.3):

\[
\text{Costs} = 9.9 + \frac{1}{|\text{PPD}|} \quad (4)
\]

and suppose for simplicity that constraints are satisfied for all variables considered, the following optimal solution is achieved:

\[
\text{Costs}^0 = 10 \text{ k£}
\]

\[
\text{PPD}^0 = \pm 10\%
\]

Practically, the dependence of Costs on PPD is more complex and includes building and plant models often it is dynamic problem [4-7]. Variables are usually control parameters of the plant (supply air temperature, mass flow rate etc.). However, the nature of the problem is the same. The only difference is that instead to be a fuzzy mathematical problem it became fuzzy optimal control problem or fuzzy optimization problem in general.

If apply conventional optimization problem (1) comfort characteristics will be in the required range, but on one of its marginal values (+ or -10% dissatisfied).

Suppose that comfort is described with relatively strong membership function:
Then the following fuzzy multiobjective optimization problem arise:

\[
\mu_{\text{result}} \rightarrow \max
\]

s.t. \( \mu_{\text{result}} = \mu_{\text{costs}} \AND \mu_{\text{comfort}} \AND \mu_{\text{constraints}} \)

Its solution could be easily found analytically [3] from:

\[
\mu_{\text{costs}} (\text{PPD}) = \mu_{\text{comfort}} (\text{PPD}) \quad (8a)
\]

\[
\text{PPD}^2+10|\text{PPD}|-100 = 0 \quad (8b)
\]

It is supposed for simplicity that constraints are satisfied for all values of variables considered. The fuzzy optimal solution is:

\[
\text{Costs}^* = 10.06 \text{ k£ \ (-0.6\%)}
\]

\[
\text{PPD}^* = \pm 6.2\% \ (+38\%)
\]

It is seen that significant improvement of the comfort requirements (with 38%) could be achieved with slightly higher costs (higher with only 0.6%). For the more complex (and realistic) fuzzy optimization problems the solution could be found numerically [18].

4 Conclusions

Optimization of building thermal systems is treated in the paper in the framework of fuzzy mathematical programming. This new approach allows to formulate more precisely the problem which compromises energy saving and thermal comfort satisfaction under given constraints. This fuzzy optimization problem is solved analytically under some assumptions.

Fuzzy approach to optimization of building thermal systems allow us to formulate problem more flexibly, user-friendly and to analyze more
precisely and deeply the whole range of possible solutions. It help us to avoid necessity of scaling and normalization applied in other approaches (when penalty functions are used) as well as infeasible solutions (infeasible solutions in fuzzy optimization problem has degree of acceptance 0 and are not considered). The problem is formulated in more natural multiobjective fashion and is solved efficiently either numerically either analytically in some cases.

An example illustrates the viability of the approach proposed. A solution which significantly (with 38%) improves comfort is found which is more energetically expensive with only 0.6%.

5 References
