Is the Big Rip unreachable?

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A R T I C L E   I N F O

Article history:
Received 15 July 2018
Received in revised form 2 August 2018
Accepted 3 August 2018
Available online 24 August 2018
Editor: M. Trodden

A B S T R A C T

I investigate the repercussions of particle production when the Universe is dominated by a hypothetical phantom substance. I show that backreaction due to particle production prevents the density from shooting to infinity at a Big Rip, but instead forces it to stabilise at a large constant value. Afterwards there is a period of de-Sitter inflation. I speculate that this might lead to a cyclic Universe.

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A phantom substance is defined as a fluid which violates the null energy condition because its pressure is $p < -\rho$, where $\rho$ is its density. Such a hypothetical substance is also called exotic matter, and it is necessary for keeping a wormhole traversable. In cosmology, if the Universe content is dominated by phantom density, then the latter increases and at some finite time $t_{\text{max}}$ it becomes infinite, in a singularity called the Big Rip. This is because the Universe undergoes super-inflation where the accelerated expansion gives rise to an event horizon, whose dimensions shrink to zero at $t_{\text{max}}$, ripping apart all structures, from galaxies to atoms.

But is it so? So far, most of the studies of the dynamics of the Universe when undergoing accelerated expansion have ignored particle production due to the event horizon, because it is usually negligible, in inflation for example. In this work, it is argued that particle production cannot be ignored when the accelerated expansion is driven by phantom density. Backreaction due to particle production renders the Big Rip singularity unreachable.

We work in Einstein gravity only. In the following, we use natural units, where $c = \hbar = 1$ and Newton’s gravitational constant is $8\pi G = m_p^2$, with $m_p = 2.43 \times 10^{18}$ GeV being the reduced Planck mass. For simplicity, isotropy and spatial flatness is assumed throughout.

The existence of an event horizon during accelerated expansion results in particle production of all light (meaning with mass smaller than the Hubble rate $m < H$) non-conformally invariant fields (e.g. a light scalar field, such as the inflaton). During quasi-de Sitter inflation, particle production results in the gravitational generation of density of the order of thermal density with temperature the Hawking temperature of de Sitter space $T = H/2\pi$ [1–7]

$$\rho_{\text{tr}} = \frac{q}{30} g_\text{eff}^c (\frac{H}{2\pi})^4, \quad (1)$$

where $g_\text{eff}^c$ is the number of effective relativistic degrees of freedom which undergo particle production (i.e. are light and non-conformally invariant) and $q \sim 1$ is some constant factor due to the fact that the resulting $\rho_{\text{tr}}$ is not actually thermal.

Thus, the Friedmann equation obtains an additional term of the form

$$H^2 = \frac{8\pi G}{3} \rho + CH^4, \quad (2)$$

where $\rho$ is the density of the substance which causes the accelerated expansion and, in view of Eq. (1), we have

$$CH^4 = \frac{8\pi G}{3} \rho_{\text{tr}} \Rightarrow C = \frac{qG}{180\pi} g_\text{eff}^c. \quad (3)$$

Strictly speaking, the above are true for $\rho, H \simeq \text{constant}$, which results in quasi-de Sitter inflation. In this case, the $CH^4$ contribution in the Friedmann equation (2) is negligible and is usually ignored (but see Ref. [9]). However, one expects the same phenomenon to take place in any kind of accelerated expansion, since gravitational particle production occurs whenever there is an event horizon (as with black hole radiation, for example). Thus, qualitatively, the same source term $CH^4$ would appear in the Friedmann equation as $H$ sets the scale of the accelerated expansion and

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1 Note that, particle production is a non-equilibrium effect and in an isotropic universe its classical counterpart is bulk viscosity [8].
$C \propto g^{\mu\nu}$ as in Eq. (3). This term would become important if the cause of accelerated expansion is a phantom substance.\(^2\)

If \( \rho \) is the density of a phantom substance then the Friedmann equation (ignoring the extra term CH\(^4\), because it can be negligible at first) results in \( \dot{\rho} > 0 \) and \( \dot{H} > 0 \). This results in super-inflation which leads to the Big Rip singularity when \( \rho \to \infty \) in finite time. However, the growth of \( H \) means that the CH\(^4\) in the Friedmann equation (2) will eventually become important, because it increases faster than the \( H^2 \) term. This may halt the growth of \( \rho \) and prevent the Big Rip from happening \([12,13]\).

The solution to Eq. (2) is
\[
H^2 = \frac{1}{2C} \left( 1 + \sqrt{1 - \frac{32\pi G C}{3} \rho} \right). \tag{4}
\]

For small density, the above equation results in either \( H^2 = 1/C = \) constant or \( H^2 = (8\pi G/3)\rho \), which is the usual Friedmann equation. The latter solution corresponds to the negative sign in the brackets. However, the above also shows that there is a maximum possible value of \( \rho \), which is \( \rho_{\text{max}} = 3/32\pi G C \), in which case there is a maximum value of the Hubble rate \( H_{\text{max}} = 1/\sqrt{2C} \). This means that the Big Rip is unreachable, prevented by the backreaction of the gravitational particle production.\(^4\)

Using Eq. (3), we can obtain an estimate of the maximum density
\[
\rho_{\text{max}} = \frac{3}{32\pi G C} = \frac{135}{8G^2} g^{\mu\nu} = \frac{3H_{\text{max}}^2}{16\pi G}
\]
\[
\Rightarrow \rho_{\text{max}} = \sqrt{g^{\mu\nu}} \left( \frac{30}{8G^2} \right)^{1/4} m_p. \tag{5}
\]

Thus, \( \rho_{\text{max}}^{1/4} \) can be smaller than the Planck scale only when \( g^{\mu\nu} \) is very large. Considering string theory, we can reduce \( \rho_{\text{max}}^{1/4} \) to the string scale \( \sim 10^{17} \text{GeV} \) by considering \( g^{\mu\nu} \sim 10^5 \) or so. We assume that \( \rho_{\text{max}}^{1/4} < m_p \) such that quantum gravity considerations can be ignored.\(^5\)

Exactly how the density evolves can be revealed by the study of the continuity equation
\[
\dot{\rho} + 3(1 + w)H\rho = 0, \tag{6}
\]
where \( w = p/\rho < -1 \) is the barotropic parameter of the phantom substance. For simplicity, we consider \( w = constant \). Using Eq. (4) with the negative sign in the brackets, Eq. (6) can be analytically solved to give
\[
\frac{1}{\sqrt{2}} \ln \left( \frac{\sqrt{2} + \sqrt{2C} H \sqrt{2} - 1}{\sqrt{2} - \sqrt{2C} H \sqrt{2} + 1} \right) = 2 \left( \frac{1}{\sqrt{2C} H} \right) (t_{\text{max}} - t), \tag{7}
\]
where \( t_{\text{max}} \) corresponds to the time when \( \rho = \rho_{\text{max}} \). Note that Eq. (4) (with the negative sign in the brackets) implies that \( \sqrt{2C} H = (1 - \sqrt{1 - u})^{1/2} \), where we have defined \( u = \rho/\rho_{\text{max}} \) i.e. \( u(t_{\text{max}}) = 1 \).

When \( H < H_{\text{max}} = 1/\sqrt{2C} \), the last term in the left-hand-side of the above dominates and we find \( H^{-1} = \frac{1}{2} [1 + w/(t_{\text{max}} - t)] \) as with standard phantom dark energy, only the time \( t_{\text{max}} \) does not denote the Big Rip, i.e. \( \rho(t_{\text{max}}) \to \infty \), but instead we have \( \rho(t_{\text{max}}) = \rho_{\text{max}} \) which is finite and given by Eq. (5).

The solution in Eq. (7) only applies for \( t \leq t_{\text{max}} \). At \( t_{\text{max}} \) the solution becomes zero.\(^6\) We can investigate what happens when \( t > t_{\text{max}} \) by considering the continuity equation (6), which gives
\[
\dot{u} = 31 + w/Hu \Rightarrow \dot{u}_{\text{max}} = \frac{3[1 + w]}{\sqrt{2C}} > 0, \tag{8}
\]
where \( \dot{u}_{\text{max}} \equiv \dot{u}(t_{\text{max}}) \) and we used that \( H_{\text{max}} = 1/\sqrt{2C} \) and \( u(t_{\text{max}}) = 1 \). Thus, there is a tendency for the density to become larger than \( \rho_{\text{max}} \), which, however, is forbidden by Eq. (4). This is really because Eq. (2) is not valid any more. Indeed, for \( \rho > \rho_{\text{max}} \), the density of the gravitationally produced particles would be larger than the phantom density itself, which cannot happen (where is the energy coming from?).

This suggests that the continuity equation needs augmenting, since the removal of energy by gravitational particle production is not considered in Eq. (8); it is implicitly assumed negligible. To take this into account we introduce a negative source term on the right-hand-side of Eq. (6), which becomes (see also Ref. [15])
\[
\dot{\rho} - 3(1 + w)H\rho = -3[1 + w/H\rho_{\text{ge}}] - \frac{9}{8G} 1 + w/C H^5. \tag{9}
\]

The form of this term is given by \( \delta \rho/\delta t \), where \( \delta \rho = -\rho_{\text{ge}} \propto H^4 \) as in Eq. (1), and \( \dot{t} \sim H^{-1} \) is the Hubble time. This is because the relativistic particles produced gravitationally in a Hubble time are diluted by the accelerated expansion and replenished by the particles produced in the following Hubble time, meaning that \( \delta \rho \propto H^4 \) is produced gravitationally per Hubble time \( \dot{t} \sim H^{-1} \). The precise value is \( \delta t = H^{-1}/3[1 + w] \), which results in \( u_{\text{max}} = 0 \).

Combining Eqs. (4) and (9) we may write the continuity equation as
\[
\dot{u} = 6[1 + w/H_{\text{max}}\sqrt{1 - \dot{u}(1 - \sqrt{1 - u})^{3/2}}, \tag{10}
\]
where \( H_{\text{max}} = 1/\sqrt{2C} \). From the above it is evident that \( u_{\text{max}} = 0 \) as expected. Also note that, in the limit \( u \ll 1 \) the above becomes
\[
\dot{u} \approx \frac{1}{2} [1 + w/H_{\text{max}}u^{3/2}] \equiv 3[1 + w/Hu], \tag{11}
\]
which is the usual continuity equation (cf. Eq. (8)) and we considered that \( H = H_{\text{max}}(1 - \sqrt{1 - u})^{-1/2} \approx H_{\text{max}} u^{1/2} / \sqrt{2C} \) in this limit. Note that in the limit \( u \ll 1 \) we recover the Friedmann equation as expected because \( H^2 \approx 1/2 t_{\text{max}}^2 \). Here \( u = 8\pi G \rho / 3 \), where \( H_{\text{max}} = 1/\sqrt{2C} \) and \( \rho_{\text{max}} \) is given by Eq. (5).

\(^2\) In general, in accelerated expansion the Hawking temperature is \( T = \frac{1}{2} (3w + 1)/(H/2m) \).\(^10\) The numerical factor \( \frac{1}{2} (3w + 1) \) can be incorporated into \( g \) in Eq. (1).

\(^3\) In principle, other terms involving the derivatives of the Hubble rate, such as \( H\dot{H}, H^2 \dot{H} \) or \( H^3 \) may also be important, additionally to the \( H^2 \) term considered here. Such terms are negligible in quasi-de Sitter inflation because \( |H| \ll H^2 \), but this would not necessarily be so for a phantom substance. However, in Ref. [11] it is shown that the density of such derivative terms is proportional to \( \alpha \), whereas the latter is the coefficient of \( R^2 \) in a modified gravity Lagrangian \( L = 2m^2 GR + a H^2 \). In this work we consider Einstein gravity only, which means \( a = 0 \) and these terms are absent.

\(^4\) A similar result was obtained in Ref. [14], where the effect of the conformal anomaly was taken into account, which can also produce a contribution \( \delta \rho \propto H^4 \).

\(^5\) In a quantum gravity setup, the \( H^4 \) correction in the Friedmann equation (2) might be the leading order to terms proportional to \( H^6 \) or \( H^8 \) for example. Such terms would grow even faster than the \( H^4 \). However, we expect such terms to be Planck suppressed such that they always remain subdominant because \( 4^n/m_p^4 < H^4 \) for all \( n \geq 1 \) since \( H \leq H_{\text{max}} \ll m_p \) because \( \rho_{\text{max}}^{1/4} < m_p \).

\(^6\) In view of Eqs. (1) and (5), we have \( \rho_{\text{ge}} \leq \rho \equiv 8\pi G \rho / 3 \geq CH^4 \). This means that we cannot connect with the positive branch of \( H^2 \) in Eq. (4), which ranges between \( 1/\sqrt{2C} \) and \( 1/\sqrt{C} \), because this requires \( 8\pi G \rho / 3 < CH^4 \).
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braneworlds

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then

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dramatically

where,

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Eq. (5))
happens

backreaction

If

Fig. 1.

134


This

7 Note

that

the

Planck

satellite

observations

favour

phantom

dark

energy

[16].

8 Of

course,

we

know

that

the

observed

red

spectrum

of
curvature

perturbations

demands

that

ρ

is

not

zero,

but

slightly

negative

during

inflation.

One

might

hypothesise

that

the

dramatic

decay

of

ρ

at

the

end

of

inflation

is

precipitated

by

some

greatly

suppressed

decay

process

during

inflation.

An

example

of

this

possibility

is

studied

in

Ref.
[17].

9 This

is

similar

to

Ref.
[18] although

in

those

works

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is

taken

to

vary

periodically

in

time.

For

a

cyclic

Universe

due

to

phantom

dark

energy

in

the

cycle

of

braneworlds

see

Ref.
[19].

substantially

altered.

In

many

cyclic

models,

for

example

the

Mixmaster

universe

in

Ref.
[21],

the

cumulative

anisotropic

effects

grow

and

become

overwhelming,

possibly

destabilising

the

cyclic

behaviour.

In

contrast

to

the

Mixmaster

scenario

however

(and

other

cyclic

models,

e.g.

Ref.
[22]),

the

cyclic

universe

considered

here

does

not

involve

a

contracting

phase

(which

would

enlarge

the

anisotropy).

The

total

density

of

the

universe

is

growing

and

falling

but

the

expansion

never

halts

or

reverses

itself.

Thus,

in

this

case,

any

existing

anisotropy

might

remain

negligible,

although

this

needs

to

be

investigated.

We

have

discussed

the

repercussions

of

particle

production

when

the

Universe

is

dominated

by

a

hypothetical

phantom

substance.

We

have

argued

that

backreaction

due
to

particle

production

prevents

the

density

from

shooting

to

infinity

at

a

Big

Rip,

but

instead

forces

it

to

stabilise

at

a

large

constant

value,

which

could

be

near

the

string

scale

or

the

scale

of

grand

unification,

which

is

a

little

lower.

After

assuming

its

maximum

value

there

is

a

period

of

de-Sitter

inflation.

This

might

lead
to

a

cyclic

Universe,

provided

some

unknown

mechanism

terminates

inflation

and

dramatically

reduces

the

phantom

density,

giving

rise

to

the

thermal

bath

of

the

hot

big

bang.

We

have

considered

Einstein

gravity

and

ignored

quantum

gravity

corrections

taking

the

maximum

density

to

remain

sub-Planckian.10 We

have

not

introduced

a

specific

model

of

phantom

dark

energy,11 in

order

to

emphasise

that

our

treatment

and

results

are

generic

and
due
to

the

existence

of

an

event

horizon

in

accelerated

expansion,

which

leads

to

particle

production

as

in

black

holes.

In

that

sense,

our

finding

that

the

Big

Rip

is

unreachable,

seems

unavoidable.

Acknowledgements

I

would

like
to

thank

P.

Anderson

and

G.

Rigopoulos

for

discussions.

My

research

is

supported

(in

part)

by

the

Lancaster–Manchester–Sheffield

Consortium

for

Fundamental

Physics

under

STFC

grant:

ST/L000520/1.

References


10 For avoiding

the

Big

Rip

singularity

in

modified

gravity

see

Ref. [23].

11 We

only

took

w = constant < −1 for simplicity.